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*The George E Brown, Jr. Network for Earthquake Engineering Simulation*

# Using the Unscented Kalman Filter in Fast Hybrid Testing

by

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# Contents



**Abstract**

In this document the Kalman Filter is described as an algorithm to estimate the state of an unknown system. Extensions to the standard linear Kalman Filter are also described: the Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF). Both the EKF and UKF can be used to estimate nonlinear systems to various degrees, at the cost of more computational time. The theory and basic update computations of all three filters are shown, and the process of estimating not only state but parameters is described.

An example hybrid test using the Kalman Filter is described, and an appropriate model is devised using both state and parameter variables. The filter is used to process a noisy displacement measurement and produce estimates of displacement, velocity and amplifier gains. The resulting displacement waveforms show a reduction in noise amplitude and controller instability. In addition, the filter is shown to be capable of estimating system parameters during operation; in this case, the filter proved capable of following changes in amplifier gain as a user modified the gain manually during a hybrid test.

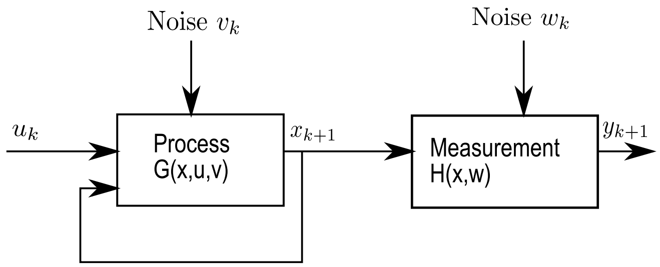


Figure 1: Basic state and noise model used for the Kalman Filter.

# 1 Background and Motivation

When running a hybrid test, especially a real-time hybrid test, one concern is that of interacting with the physical specimen using an actuator, and properly controlling that actuator based on noisy measurement data. When attempting to control the actuator using a noisy measurement, problems can occur with accuracy and stability. These problems are exacerbated by the high-speed control requirements of a real-time hybrid test.

A commonly-used solution is to estimate the state from noisy measurements using a Kalman Filter [?]. The Kalman Filter is a recursive filter implementation that uses successive noisy measurements to estimate state variables. If the noise is white gaussian noise and the system is linear, the Kalman Filter is proven to converge to the optimal state estimate. Coupled with multivariate state-space modeling of the actuator/specimen, the Kalman Filter can be a powerful tool for controlling noisy or uncertain systems.

## 1.1 Definitions and Concepts

The Kalman Filter assumes a state-space model of the system being estimated. This state-space model is generally of the form

$$\begin{aligned} x_{k+1} &= f(x_k, u_k, v_k) \\ y_k &= h(x_k, w_k) \end{aligned}$$

Where $x_k$ is the process state, $y_k$ is the measurement, $u_k$ is the input, and $v_k, w_k$ are random gaussian noise processes. The Kalman Filter calculates $\hat{x}_k$, which is an estimate of the actual state $x_k$; in addition the error covariance $P_k$ is computed, which describes the accuracy (or lack of accuracy) of the state estimate $\hat{x}_k$.

At each timestep $k$, the Kalman Filter updates the estimate $\hat{x}_k$ from the previous estimate $\hat{x}_k$ and the input $u_k$. This first computational step is referred to here as the "time update" and produces the result $\hat{x}_k^-$. The second computational step combines the estimate $\hat{x}_k^-$ and the measurement $y_k$ to produce an improved estimate $\hat{x}_k^+$. The notations $(.)^-$ and $(.)^+$ refer to quantities before and after this second step, call the "measurement update."

## 1.2 The Standard Kalman Filter

For a linear system, the process state space model is

$$\begin{aligned} x_{k+1} &= F_k x_k + B_k u_k + v_k \\ y_k &= H_k x_k + w_k \end{aligned}$$

thus the Kalman update states for a linear model become:

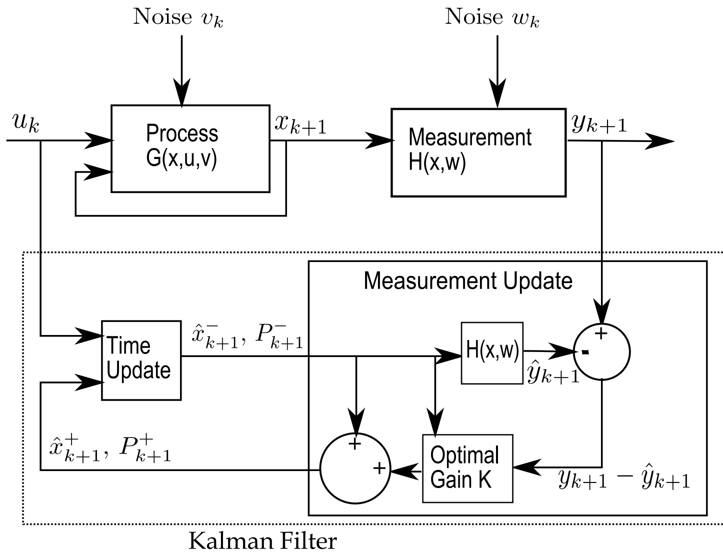


Figure 2: The Kalman Filter uses input ($u_k$) and measurement ($y_k$) data a to update a state estimate $\hat{x}_k$ and error covariance $P_k$.

$$
\begin{aligned}
& \text{Time Update} \\
\hat{x}_k^- &= F_k \hat{x}_{k-1}^+ + B_{k-1} u_{k-1} \\
\hat{y}_k^- &= H_k \hat{x}_k^- \\
P_k^- &= F_k P_{k-1}^+ F_k^T + Q_k \\
& \text{Measurement Update} \\
K_k &= P_k^- H_k^T \left[ H_k P_k^- H_k^T + R_k \right]^{-1} \\
\hat{x}_k^+ &= \hat{x}_k^- + K_k \left[ y_k - \hat{y}_k^- ) \right] \\
P_k^+ &= P_k^- - K_k H_k P_k^-
\end{aligned}
$$

As mentioned before, the superscripts $^-$ and $^+$ indicate state quantities $x_k$ and $P_k$ computed before $(-)$ and after $(+)$ the measurement $y_k$ is taken into account.

## 1.3 The Extended Kalman Filter

For a nonlinear system, the more general case of nonlinear functions is assumed:

$$
\begin{aligned}
x_{k+1} &= f(x_k, u_k, v_k) \\
y_k &= h(x_k, w_k)
\end{aligned}
$$

In order to perform the Kalman updates, the nonlinear functions $f$ and $h$ must be linearized around the current estimated state $\hat{x}_k$, producing Jacobian matrices $A, W, V, H$

$$
\begin{array}{cc}
A(i,j) = \frac{\delta f_i}{\delta x_j} & W(i,j) = \frac{\delta f_i}{\delta w_j} \\
H(i,j) = \frac{\delta h_i}{\delta x_j} & V(i,j) = \frac{\delta h_i}{\delta v_j}
\end{array} \tag{1}
$$

which are used in the update equations for the Kalman Filter:

Time Update

$$\begin{aligned}
\hat{x}_k^- &= f(\hat{x}_k, u_k, 0) \\
\hat{y}_k^- &= h(\hat{x}_k^-, 0) \\
P_k^- &= A_k P_{k-1}^+ A_k^T + W_k Q_{k-1} W_k^T
\end{aligned}$$

Measurement Update

$$\begin{aligned}
K_k &= P_k^- H_k^T \left[ H_k P_k^- H_k^T + V_k R_k V_k^T \right]^{-1} \\
\hat{x}_k^+ &= \hat{x}_k^- + K_k \left[ y_k - \hat{y}_k^- \right] \\
P_k^+ &= P_k^- - K_k H_k P_k^-
\end{aligned}$$

This form of the Kalman Filter using linearization around the current operating state is called the *Extended Kalman Filter* (EKF).

## 1.4 The Unscented Kalman Filter

The EKF linearizes the nonlinear functions $f$ and $h$ using Jacobian matrices, effectively producing a first order approximation to the nonlinear functions. For highly nonlinear functions, this approximation can produce poor estimates or even divergence in the estimate $\hat{x}_k$. To produce a better approximation, the *Unscented Kalman Filter* (UKF) [**?**] uses the Unscented Transform to provide a second-order approximation of the nonlinear functions.

The Unscented Transform uses augmented state and covariance variables which include the noise variables $v_k$ and $n_k$,

$$\begin{aligned}
x_k^a &= \begin{bmatrix} \hat{x}_k \\ v_k \\ n_k \end{bmatrix} \\
P_k^a &= \begin{bmatrix} P_k & 0 & 0 \\ 0 & P_v & 0 \\ 0 & 0 & P_n \end{bmatrix}
\end{aligned}$$

These augmented variable are used to generate sigma points $\chi_{i,k}$ around the original point $\hat{x}_k$; these sigma points then passed through the nonlinear functions $f$ and $h$ and summed on the other side [**?**, pp. 448-450]:

Generate Sigma Points:

$$\begin{aligned}
\chi_{i,k-1}^a &= \begin{bmatrix} \chi_{i,k-1}^x \\ \chi_{i,k-1}^v \\ \chi_{i,k-1}^n \end{bmatrix} && \forall i = 1, \ldots, 2L \\
\chi_{i,k-1}^a &= \hat{x}_{k-1}^a + \left( \sqrt{n P_{k-1}^a} \right)_i && i = 1, \ldots, L \\
\chi_{i+L,k-1}^a &= \hat{x}_{k-1}^a - \left( \sqrt{(n P_{k-1}^a} \right)_i^T && i = 1, \ldots, L \\
W_i &= \tfrac{1}{2L} && i = 1, \ldots, 2L
\end{aligned}$$

Pass Through nonlinear functions:

$$\begin{aligned}
\chi_{i,k}^x &= f(\chi_{i,k-1}^x, \chi_{i,k-1}^v) \\
\psi_{i,k} &= h(\chi_{i,k-1}^x, \chi_{i,k-1}^n)
\end{aligned}$$

Time Update:

$$
\begin{aligned}
\hat{x}_k^- &= \sum_{i=0}^{2L} W_i^m \chi_{i,k}^x \\
\hat{y}_k^- &= \sum_{i=0}^{2L} W_i^m \psi_{i,k} \\
x'_{i,k} &= \chi_{i,k}^x - \hat{x}_k^- \\
y'_{i,k} &= \psi_{i,k} - \hat{y}_k^- \\
P_k^- &= \sum_{i=0}^{2L} W_i^c (x'_{i,k})(x'_{i,k})^T
\end{aligned}
$$

Measurement Update:

$$
\begin{aligned}
K_k &= \left[\sum_{i=1}^{2L} W_i (x'_{i,k})(y'_{i,k})^T\right] \left[\sum_{i=1}^{2L} W_i (y'_{i,k})(y'_{i,k})^T\right]^{-1} \\
\hat{x}_k^+ &= \hat{x}_k^- + K_k \left[y_k - \hat{y}_k^-\right] \\
P_k^+ &= P_k^- - K_k \left[\sum_{i=1}^{2L} W_i (y'_{i,k})(y'_{i,k})^T\right] K_k^T
\end{aligned}
$$

The resulting filter correctly estimates the mean and covariance to third order, compared to the first-order accurate estimate of the EKF produced by linearizing around the estimated state. Thus the main advantage of the UKF is that it produces a higher-order approximation of the nonlinear functions involved in the process model.

# 2 Test Setup and Details

To examine the filtering and parameter estimation properties of the UKF, hybrid tests were performed using the Desktop Hybrid Platform (figure 3, running the Mercury analysis software coupled to instrumentation using the LabView environment.

The hybrid test used here involves coupling a physical specimen with numerical analysis. The analysis is a simple two-truss model, and the physical specimen is a brass bar clamped to metal brackets.

## 2.1 Mercury Analysis Software

The *Mercury* analysis software [?] is used for the desktop system. *Mercury* is software produced at CU-Boulder specifically for use in hybrid tests, and can run either as a standalone program in Microsoft Windows or as a software module embedded within Real-Time applications such as Simulink or Real-Time LabView. The intent is to allow users to prototype and test out their analysis models in a standard Windows machine; once the model is working as intended, it can be moved over to the real-time system with little or no change and used for a hybrid test.

In order to use *Mercury* with the desktop system, the *Mercury* software was embedded within a LabView module which is executed on a PC running Real-Time LabView. Running the analysis inside a module using Real-Time LabView provides the determinism needed to satisfy hard real-time requirements. The *Mercury* LabView module (see figure 4) provides

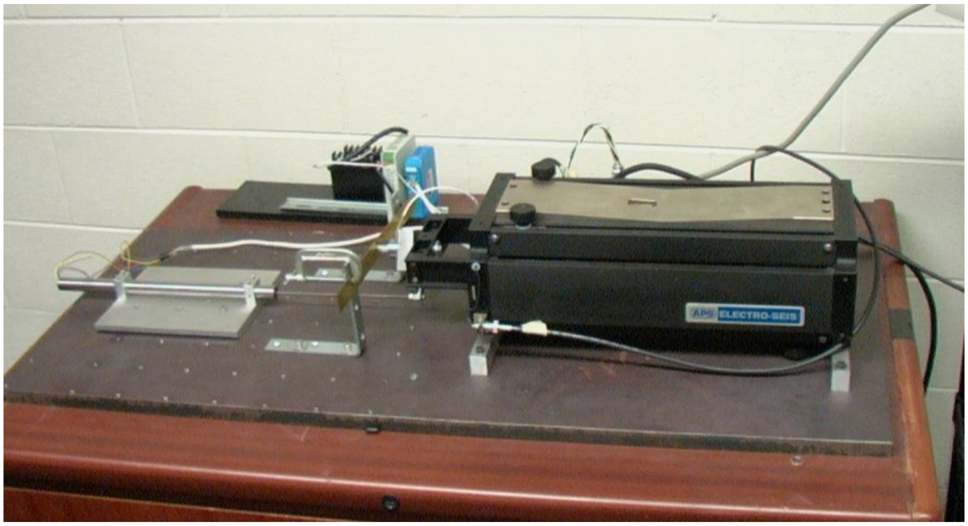


Figure 3: The desktop hybrid platform, used for prototyping and testing Kalman Filter configurations.

input and output ports which can be connected up to the appropriate instrumentation modules, as detailed in the next section.

### 2.2 Hardware Integration and Use

The hardware used to interact with a physical specimen is an *Electro-Sies* electromagnetic shaker for exerting force on the specimen, along with an LVDT and load cell for measurement displacement and force of the specimen. These components are all connected to a desktop PC running Real-Time LabView, shown in figure 3. The instrumentation can be accessed in LabView using the DAQmx modules, which allow reading and writing of data to and from the instruments by simply connecting wires between modules. The *Mercury* software is embedded in Real-Time LabView and connected up to various I/O modules which allow it to drive the actuator and receive measurement values from the sensors, shown in figure 4. The hybrid integration method devised by Shing [?] is used to drive the specimen and incorporate measurement data back into the computational analysis.

The mounting plate and brackets are used to allow the attachment of various physical specimens; the specimen attached in figure 3 is a flexible brass bar used for validation testing. Other specimens can be attached and detached using standard nuts and bolts in a matter of minutes, facilitating quick testing and prototyping of various specimens.

The control signal to the actuator is generated by a standard PID controller using the displacement value from the analysis as the current set point. The measured displacement used by the PID controller can be used directly or via a Kalman Filter as shown in figures 5 and 6. In section 3.3 we examine the properties of the controller with and without a Kalman Filter inserted into the control loop.

## 3 Actuator State and Parameter Estimation

In order for the Kalman Filter to function it requires a model of the system that is being estimated. In our case, the system should represent the actuator/specimen dynamics as well as the variable gain present in the power amplifier. To this end, we build a fairly simple system with three state variables and two parameters to be estimated.

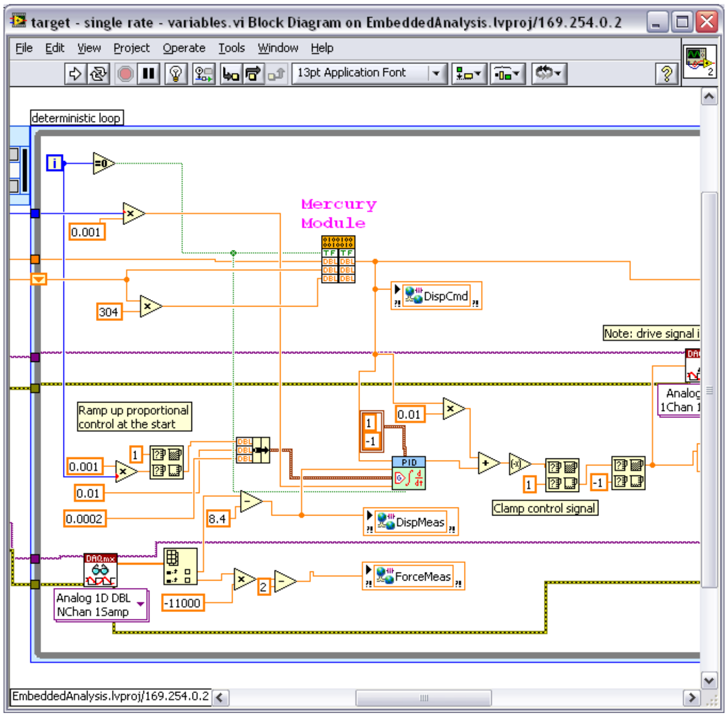


Figure 4: Mercury module embedded within a LabView program. The Mercury analysis program is completely enclosed by the small rectangle labelled "Mercury Module"

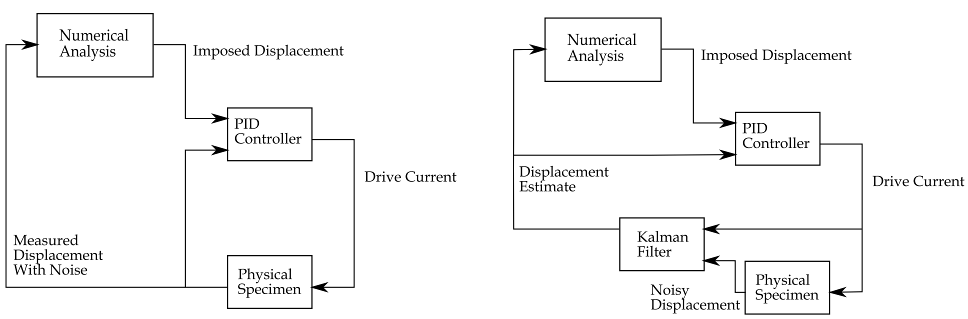


Figure 5: Data flow of a Fast Hybrid Test using a PID controller to control actuator position, with and without a Kalman Filter estimating the actuator state.

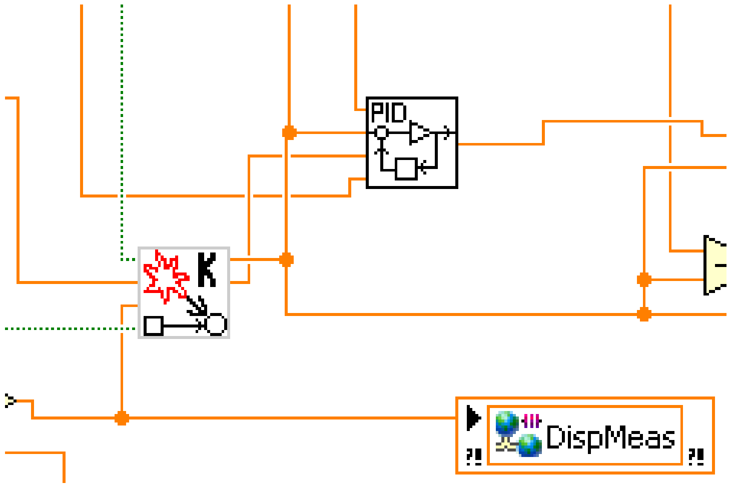


Figure 6: Snapshot of a section of the LabView block diagram, showing the Kalman Filter (marked "K") inserted just before the PID controller.

The actual dynamics of an electromagnetic actuator are more complex than the simple model presented here, and ideally a model with more state variables and parameters would conceivably produce a more accurate state estimate. However, higher-order models require more computation time, $O(n^3)$ with respect to the number of state variables. Using a large number of states would require prohibitively large amounts of computation, so that we could no longer run at real-time speeds. Also, the state estimate is constanly being updated and revised based on measurements made from the real system, so any small inaccuracies in the model are corrected during the measurement update step of the Kalman Filter. As long as the computer model used by the Kalman Filter is "close" to the actual system in dynamics, the state estiate of the filter will converge to the actual state over time using successive measurement values.

## 3.1 State Estimation

The actuator/specimen is modeled as a simple second-order system. The key phenomena we wish to model are the relationship between position, velocity, and acceleration. The state variables used here are:

- $p_k$ actuator position from center, in inches
- $\dot{p}_k$ actuator velocity, in inches/second
- $I_k$ electrical current in the electromagnetic actuator, producing a force (and thus acceleration) of the actuator

The state and process model equations use for the actuator are:

$$x_k = \begin{bmatrix} p_k \\ \dot{p}_k \\ I_k \end{bmatrix} \tag{2}$$

$$x_{k+1} = \begin{bmatrix} 1 & \Delta t & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_k + m_k \tag{3}$$

This set of equations is actually still a linear system, and we could use the standard Kalman Filter to estimate this system. However, the addition of parameter estimation produces a nonlinear system, as shown in the next section.

## 3.2 Parameter Estimation

The electrical amplifier for the actuator has an unknown gain $g_k$. We can estimate this gain by treating it as a state variable and incorporating it into the process equations:

$$\bar{x}_k = \begin{bmatrix} p_k \\ \dot{p}_k \\ i_k \\ g1_k \\ g2_k \end{bmatrix} \tag{4}$$

$$\bar{x}_{k+1} = \begin{bmatrix} 1 & \Delta t & 0 & 0 & 0 \\ 0 & 1 & \Delta t & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \bar{x}_k + \begin{bmatrix} 0 \\ g1_k \\ g2_k \\ 0 \\ 0 \end{bmatrix} \bar{u}_k + \bar{m}_k \tag{5}$$

With this addition of the parameters $g1_k$ and $g2_k$, the function $f(x_k)$ becomes a nonlinear function, and one of the nonlinear Kalman Filters would be required. In this case we choose the UKF for its higher-order accuracy.

## 3.3 Controller with and without Filtering

The controller used is a standard PID controller with an additional feedforward component. The controller parameters are tuned to minimize absolute error between command and measured displacement, and then a hybrid test is run. During the hybrid test, we examine the effect that the Kalman Filter has on the control signal as well as the stability of the overall system.

The initial test is performed without any excitation; thus, the command and measured displacement values of the specimen should be equal or near to zero. In thus test the main error is due to measurement noise of the instrumentation. The use of a Kalman Filter here serves mainly to removed most–although not all–of the noise in the measured displacement. Figure 7 shows the measured displacement of this test with and without the Kalman Filter.

In the second test, the control gains of the PID controller are increased to near-unstable levels, inducing heavy "chattering" in the actuator. Inducing this near-instability provides a better contrast between the filtered and unfiltered results. As the simulation runs, the Kalman Filter is alternately turned on and off. With the filter enabled, chattering is reduced but not eliminated (figure 8); extended tuning and a more complex model used in the filter and produce further improvement. However, diminishing returns dictate that further complexity would greatly increase the computational requirement of the Kalman Filter while only slightly improving the state estiamte. Thus the model was kept at its current size: three state variables and two parameters.

In order to demonstrate the parameter estimation capabilites of the Kalman Filter used here (the UKF) a test is performed where the user manually changes the amplifier gain during the test via a physical knob on the front panel of the amplifier. Changing the amplifier gain modifies the dynamics of the entire actuator/specimen system, and the model used by the Kalman Filter must be revised to reflect the new dynamics. Using the UKF, parameters representing the amplifier gain are continually revised as the simulation runs, based off of measurement data. The plot in figure 9 shows the evolution of the parameter estimates in the Kalman Filter model as the user changes the amplifier gain, ramping the gain up and down.

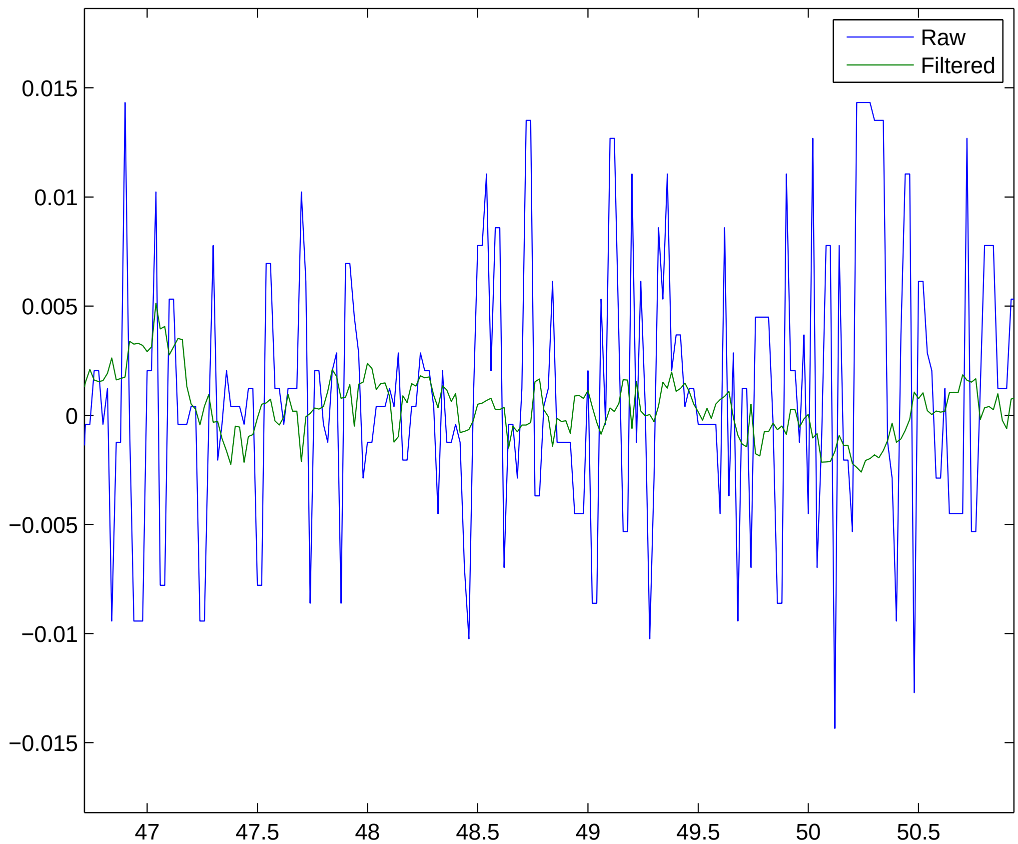


Figure 7: Comparison of the unfiltered and filtered displacement values when the hybrid test is running and “at rest” (i.e., excitation is zero)

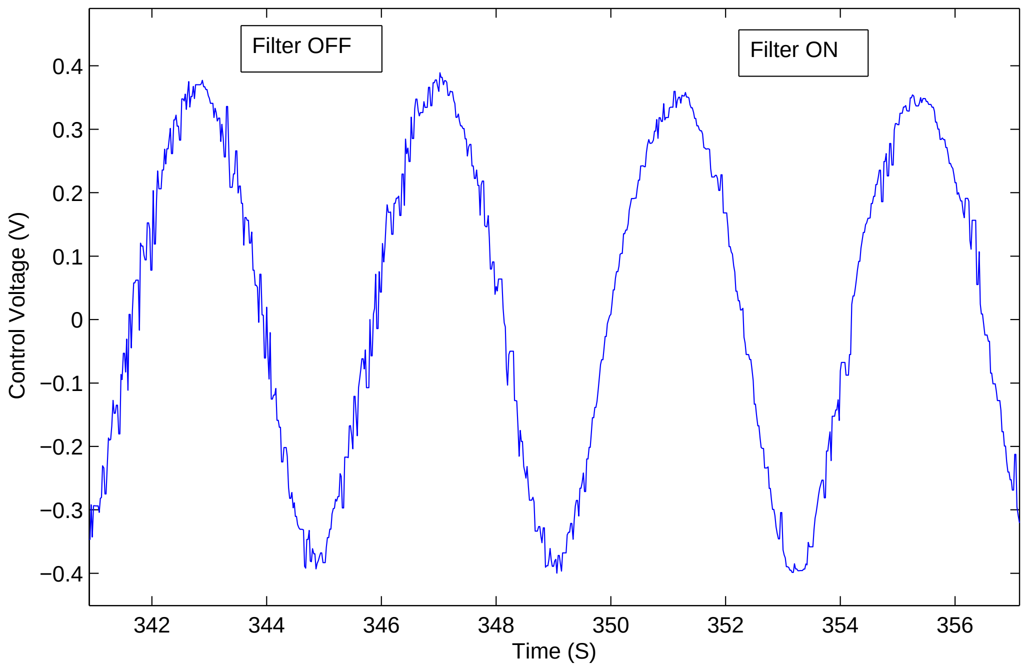


Figure 8: Time-domain data of the control signal for a hybrid test including a slow sine wave excitation. Controller gains are increased to induce unstable "chattering." The Kalman Filter is off in the first two cycles then turned on the following two cycles. The spurious oscillations that occur are significantly reduced by filtering.

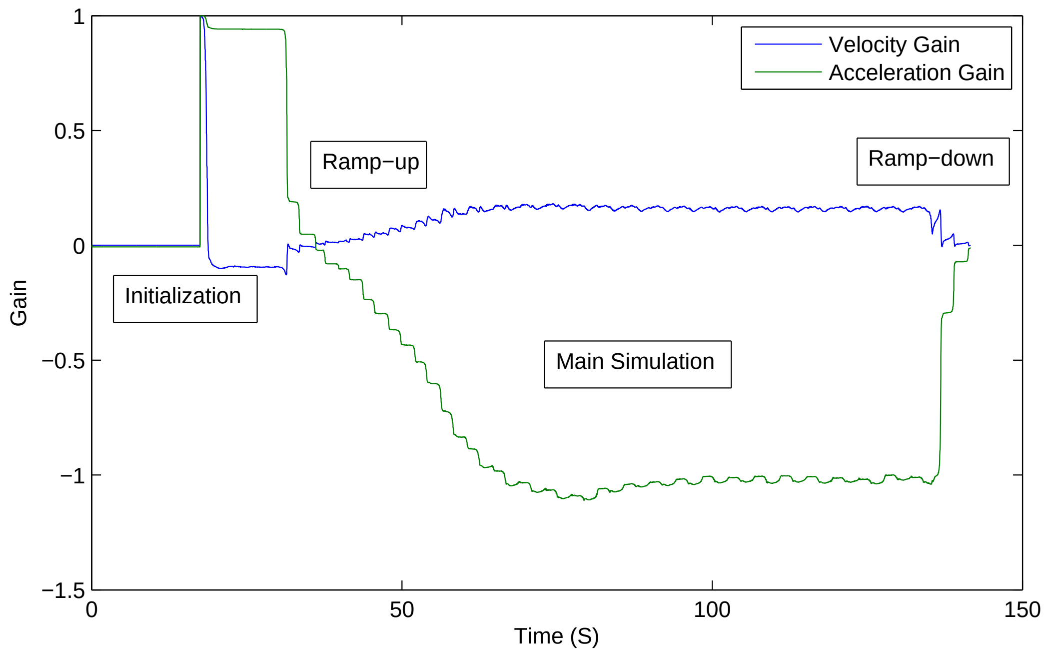


Figure 9: Online gain estimates. As the test runs, the user manually adjusts the amplifier using a physical knob on the amplifier panel. The Kalman Filter continually revises its model parameters to reflect the changing dynamics produced by varying the amplifier gain.

# 4 Conclusion

In this document the Kalman Filter is described as an algorithm to estimate the state of an unknown system. Extensions to the standard linear Kalman Filter are also described: the Extended Kalman Filter (EKF) and Unscented Kalman Filter (UKF). Both the EKF and UKF can be used to estimate nonlinear systems to various degrees, at the cost of more computational time. The theory and basic update computations of all three filters are shown, and the process of estimating not only state but parameters is described.

An example hybrid test using the Kalman Filter is described, and an appropriate model is devised using both state and parameter variables. The filter is used to process a noisy displacement measurement and produce estimates of displacement, velocity and amplifier gains. The resulting displacement waveforms show a reduction in noise amplitude and controller instability. In addition, the filter is shown to be capable of estimating system parameters during operation; in this case, the filter proved capable of following changes in amplifier gain as a user modified the gain manually during a hybrid test.

While the Kalman Filter demonstrates some ability to compensate for noisy measurement values reduce controller instabilites, the filter requires a good system model to perform well. For complex test systems, the size of the model may require considerable computation, limiting the Kalman Filters applicability in some real-time contexts. Also, the Kalman Filter can mitigate but not correct a badly designed controller or fundamentally unstable system; it can mainly be used to reduce the noisy measurements of a well-behaved system.

For future work, the Kalman filter will be extended for use in more scenarios; specifically, it can used on more complicated lab setups, such a hydraulic actuator which contains more nonlinear behavior than the EM actuator used here. The parameter estimation capabilities can also be extended to provide online estimates of the specimen stiffness as it is being tested, to detect when key events such as buckling or cracking occur.

Another intended use of the Kalman Filter is in handling the inertial forces of a test rig driven under fast actuation—in this case, more than 20 inches/second. In slower tests the force due to mass of the attachment plates and support structure can be safely ignored or neglected; at higher speeds this mass will need to be estimated and compensated for in the measurement values. Based off of initial measurements a model of the unintended inertial effects can be built and used to correct measurements during a hybrid test.

# 5 Acknowledgements

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