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Improved dynamic testing by impedance control

by

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The final copy of this thesis has been examined by the signatories, and we Find that both the content and the form meet acceptable presentation standards Of scholarly work in the above mentioned discipline. Carl, Jochen (PhD, Dept. of Civil, Environmental and Architectural Engineering) Improved dynamic testing by impedance control

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Hybrid testing combines physical testing with numerical simulation. A substructure, whose inelastic response is difficult to predict, is tested physically while the rest of the structure is simulated numerically. The computed boundary conditions for the physical substructure are applied to the structure. The corresponding response of the physical model is fed back to the numerical model which based on this feedback computes the boundary conditions for the next time interval.

The motivation of this study emerged from inaccuracies and instability problems in hybrid testing, which result from bad or unstable actuator tracking. Those problems are mainly present if the physical substructure is very stiff.

This dissertation presents an actuator control mechanism which allows for accurate and stable dynamic testing of any structure stiffness by integrating the actuatorstructure-interaction into the control design. This is achieved by controlling the actuator impedance as a function of the structure stiffness. Stability requires that the actuator does not act like a rigid but a flexible device which deforms due to the interaction force. In this study feedforward control is used to achieve accurate actuator tracking while controlling its required flexibility. The presented control model allows for stable and accurate actuator tracking for both nonlinear and super stiff structures.

DEDICATION

To my wife, sister and parents.

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1. INTRODUCTION

Hybrid testing is an experimental technique for simulating the dynamic response of structures with respect to the time domain, and combines a computational with a physical model. A substructure is tested physically while the rest of the structure is simulated in the numerical model. Hybrid testing allows for the testing of large structures and provides more accurate results than just analytical methods alone. This is because of using measured restoring forces and displacements instead of the mathematical description of the non-linear model which is very hard to predict accurately. The dynamic responses of a specimen advance in a step-by-step manner through direct integration. The data obtained from the previous time step is used to compute the responses for the current time step. The controller-actuator serves as the connecting device between the numerical model and the physical substructure (Figure 1.1). It therefore plays a significant role in regard to stability and accuracy of the test and will be in the focus of this study.



Figure 1.1: Interaction of numerical and physical model through manipulator

In current dynamic testing of structures, actuators operated in feedback control are used to apply prescribed displacements or forces to components or substructures. Depending on the properties of the structure, displacement or force control may be more appropriate. Typically, actuators are designed for good position control, and are therefore meant to be mechanically stiff systems [36]. So, when the tested structure is relatively flexible, displacement control is convenient. However in some cases, for instance when the tested structure is very stiff or has inertia, force control allows for more accurate and stable testing than displacement control. Force control has therefore been applied in advanced seismic testing techniques such as the effective force method [11, 48, 60, 61] and forms of real-time dynamic hybrid testing [41]. Due to the inherent stiffness of the actuator however, force control is very sensitive to control parameters and can lead to instability because of system uncertainties or poor design of the control gains. Different methods have been developed to overcome these problems. A dual compensation scheme has been employed by MTS [35], with a primary displacement and secondary force feedback loop. Sivaselvan et. al. [55] applied dynamic force control with a hydraulic actuator in displacement feedback by introducing a flexible spring between the actuator and the structure. In other scenarios, a combination of force and displacement control may be necessary. For example, Pan et. al. [38] have used a mixed control strategy, where each degree of freedom is controlled in one of the two modes. On the other hand, Elkhoraibi and Mosalam [34] and Tzierakis and Koumboulis [57] have adopted a strategy of switching between the control modes at the same degree of freedom, based on the instantaneous stiffness. For both control modes, the control

system commonly used is of the PID (proportional, integral, derivative) type. This design however does not explicitly take into account the interaction between the actuator and the tested structure, and therefore does not perform well when the properties of the structure change drastically during the course of a test.

In this study feedforward compensation is presented as a strategy to integrate the actuator-structure-interaction into the control design. If the displacement x of an actuator is considered as its output, then the actuator may be thought of as a two-input-one-output system, the two inputs being (i) the control input u (for example the current input to a hydraulic servovalve or the voltage input to an electromagnetic coil), and (ii) the interaction force F at the interface with the tested structure. This is shown in Figure 1.2a. The idea of feedforward compensation is to annihilate the effect of the interaction force by modifying the control input as shown in Figure 1.2b.



Figure 1.2: Compensation for actuator-structure-interaction in displacement control

Similarly, if the force *F* at the interface is considered as the output, then the actuator can again be thought of as a two-input-one-output system, this time inputs being (i) the control input *u*, and (ii) the displacement *x* (or the velocity \dot{x}) at the interface as shown in Figure 1.3a. Again, the idea is to eliminate the effect of the interaction displacement (or velocity) by modifying the control input as shown in Figure 1.3b.



Figure 1.3: Compensation for actuator-structure-interaction in force control

Thus, if the objective is to track a certain quantity at the interface, then the strategy is to use the work-conjugate of that quantity for feedforward compensation. Dimig et. al. [11] employed this idea for dynamic force control by using velocity feedforward. This study applies the strategy for displacement control when the tested structure has a large (perhaps varying) stiffness. It places the feedforward strategy in the more general framework of impedance control, and provides a methodology analyzing the stategy shown in Figure 1.2 and Figure 1.3 cannot be implemented exactly since the inverse transfer functions involved are non-causal. Approximate implementations are therefore necessary. Different such feedforward schemes will be discussed. While all schemes improve the actuator tracking capability, two of them additionally controlling the actuator impedance in response to a changing structure stiffness.

This study is presented in eight chapters:

Chapter two comprises a literature review of all earlier and currently used algorithms and testing methods in hybrid testing. The chapter explains the mentioned schemes which compensate for inaccurate actuator tracking.

Chapter three explains the dynamics of hydraulic and electromagnetic actuators using linear systems theory. Based on the transfer function of the controller-actuatorstructure system, all the stability bounds of the system are derived. It explains how the structure stiffness and the applied controller gains affect the dynamic behavior of the system and why in particular stiff structures cause a high risk of instability.

Chapter four introduces the idea of impedance control.

Chapter five derives all parameters of the used electromagnetic actuator by comparing the modeled response with the different measured frequency response functions. A close match between the modeled and the measured frequency response functions give a good confirmation about the accuracy of the derived parameters.

Chapter six designs and applies impedance control in form of feedforward schemes for the chosen actuator setup. The force derived feedforward scheme is shown as a model which achieves stable and accurate actuator tracking even for nonlinear and super stiff structures.

Chapter seven integrates feedforward into hybrid testing. The analytical results are confirmed by different laboratory tests.

Chapter eight concludes the study.

Appendix A presents different algorithms which are used in hybrid testing and illustrates their advantages and disadvantages in regard to numerical damping, accuracy and frequency dependency.

Appendix B explains different impedance control schemes used in robotics and their possible application in hybrid testing.

Appendix C summarizes all transfer functions used for the electromagnetic actuator.

2. HYBRID SIMULATION

This chapter includes a general review about hybrid testing. It presents the different algorithms used to integrate the equation of motion in the numerical substructure, explains all potential error sources in the hybrid loop and reviews all currently used error compensation schemes.

To simulate the hybrid system, the complete structural model is idealized as a discrete parameter system with a finite number of degrees of freedom. The equation of motion is shown in equation (2.1), where M, C and K are the mass, damping and stiffness matrices.

$$M\ddot{u}_{n+1} + C\dot{u}_{n+1} + Ku_{n+1} = P_{n+1}$$
(2.1)

One or more physical subsystems can be tested simultaneously. Each subsystem represents one degree of freedom, in equation (2.2) shown by the index *i*.

$$m^{i}\ddot{u}_{n+1}^{i} + c^{i}\dot{u}_{n+1}^{i} + k^{i}u_{n+1}^{i} = p_{n+1}^{i}$$
(2.2)

If, for instance, the stiffness k^i is tested physically, then it is replaced in the equation of motion by the directly measured interaction force r^i .

$$m^{i}\ddot{u}_{n+1}^{i} + c^{i}\dot{u}_{n+1}^{i} + r^{i}_{n+1} = p^{i}_{n+1}$$
(2.3)

Equations (2.2) and (2.3) are equivalent if the structure stiffness is linear and if the actuator perfectly applies the displacement u_{n+1}^{i} on the structure. The equation of motion is solved using forward marching time stepping integration algorithms. The components of a hybrid test are shown in Figure 2.1.



Figure 2.1: Components for hybrid test

The online computer integrates the equation of motion utilizing the measured work conjugate, i.e. the restoring force for a commanded displacement or the corresponding displacement in force control. For each time step, the integrator computes the target displacement or force and sends a command to the actuators. The actuator applies the boundary condition to the structure and measures the corresponding response. This resulting work conjugate is then sent back to the integrator, which based on this information consequently solves for the boundary condition of the next time step.

The displacement controlled method applies a displacement to the test structure and measures the resulting force as schematically represented in Figure 2.2. It can be easily implemented with conventional quasi-static testing equipment and with some basic knowledge on numerical time integration techniques [54]. The method is called

pseudodynamic as the structure is deformed by the actuator according to the dynamic load input of the structure.



Figure 2.2: Pseudodynamic test method (UBC/EERC 2005-02)

Hybrid testing also brings new challenges and difficulties in regard to stable and accurate testing. Errors in the numerical and physical substructure can lead to inaccuracies or instability. Figure 2.3 shows the hybrid loop in more detail with its functionalities and potential errors. The errors occur at different locations in the test loop but always affect the whole system. Different error compensation schemes are used both in the numerical and physical substructure. In the following, the different components in Figure 2.3 will be explained in detail.



Figure 2.3: Hybrid loop with all functionalities and errors

2.1. Interpolation and extrapolation

Fast update rates provide for a continuous motion of the actuator improving the speed of testing and decreasing the force relaxation of structural materials. However, this also brings new challenges in regard to solving the equation of motion within the defined time, and dealing with the inherent control error and response lag of servohydraulic and electromagnetic systems. Inter- and extrapolation can allow for a smoother actuator movement and improved tracking. In order to reduce the actuator delay it is sometimes reasonable to send a predicted, extrapolated displacement to the actuator so that the actuator displacement matches approximately the commanded displacement. Nakashima [37] published a polynomial approximation procedure for multi-degree of freedom systems using a Digital Signal Processor (DSP). Using the Lagrangean polynomial, the extrapolated displacement results from the target displacements from earlier time steps and the most recently computed target displacement. Both for the displacement and velocity, a third and fourth order interpolation was found as the most reasonable in regard to accurate results and limited processing time.

2.2. Errors

Figure 2.3 shows different error sources, which can emerge from both substructures. In the While some errors only affect the accuracy of the results, others can cause instability and often require the application of error compensation schemes.

2.2.1. Sources of errors



Figure 2.4: Errors in pseudodynamic testing

Hybrid simulations are affected by errors based on modeling, implementation techniques and experimental setup as shown in Figure 2.4. Modeling errors result from the discretization of the continuous real system, assuming a finite number of degrees of freedom and lumped masses. The accuracy of the solution therefore can depend on the number of degrees of freedom. Implementation errors occur because the time integration methods, which are used to solve the differential equation of motion, use a numerical approximation for the displacement and velocity. Experimental errors result from the displacement control of the actuator, calibration errors in the instrumentation, noise generated in the instrumentation, analog to digital converters, support movement and inconsistent actuator motion [49].

From these sources of errors, experimental errors usually have the most substantial impact on the simulation results, mostly because these errors are not known prior to testing and can be large for improperly tuned experimental setups. They can be either random or systematic in nature. For both, the rate of cumulative error growth increases with the step size Ω , which is the product of the frequency ω and the time step *h*. In other words, high frequency modes embody a higher error than low frequency modes.

2.2.2. Random experimental errors

Random errors are mostly measurement errors, i.e. the experimenter measures a different displacement and force than has been applied to the specimen in reality. Noise in the measured forces, which can be seen as random errors, can excite spurious response in the high frequency modes. Algorithms with numerical damping can be used to suppress the response of those higher modes. Solving the integrated equation of motion [7] is one way to reduce those errors as the integration of the force signal filters the noise in the measurements prior to being introduced into the numerical algorithms. In general, random errors are not as problematic as systematic errors, as they do not always build up in the same direction and therefore partially cancel each other out. For explicit methods the cumulative growth of random errors can be minimized by a small time interval h.

2.2.3. Systematic experimental errors

Systematic errors are reproducible inaccuracies, which add up in the same direction. As in pseudodynamic testing the numerical and physical models are connected with a closed loop, the errors propagate due to the repetition in each time step. If they are energy adding, the numerical result can grow indefinitely and become unstable. Because of these possible resonance-like effects [52], systematic errors are more detrimental than random errors, which produce a negligible effect on the structural response.

2.2.3.1. Actuator dynamics and delay

If the actuator is not able to track the reference displacement, then the displacement error of the actuator results in force measurements at the incorrect displacements. Consequently the measured forces are introduced into the numerical integration algorithm assuming they correspond to the target displacement. If the actuator constantly lags the reference signal, this produces a reverse hysteresis loop and this way adds energy into the system. This effect of negative damping can cause instability if it is not compensated for by enough other damping in the system. It is thus necessary to compensate for the actuator delay, but an overcompensation of the phase delay might also lead to instability. Figure 2.3 shows the current compensation methods used. While the extrapolation and prediction schemes try to decrease the actuator delay, the I-modification scheme reduces the force error and thereby the added energy into the system. Also numerical damping can compensate for the negative damping due to the actuator delay, damp out higher frequencies and stabilize a low damped system. Those schemes will be discussed further below.

2.2.4. Error propagation

How harmful an error is on the overall results depends on its propagation in the repetitive hybrid loop. The error propagation can be evaluated with error amplification factors, which can be achieved with a spectral analysis. The errors, introduced in each time step, are multiplied by the error amplification factors and add up with the previous errors to the cumulative error. So, small error amplification factors are desired. A big error amplification factor implies a poor accuracy but not automatically instability.

Chang's [5] unconditionally stable explicit pseudodynamic algorithm provides better error propagation than comparable algorithms, such as the Newmark explicit method. Likewise, the earlier presented integrated equation of motion [7] has lower error amplification factors as the original equation of motion, although qualitatively they are the same.

The earlier mentioned I-modification scheme from Combescure and Pegon [10] is another way to successfully reduce the experimental error. The force feedback is corrected by $\Delta F \sim K^{T}(r-x)$, where K^{T} is the estimated initial tangent stiffness of the structure, r the reference displacement command to the actuator and x the effectively applied displacement of the actuator. The I-modification will be used in chapter seven to compensate for the actuator delay. Combescure and Pegon stated that stability is reached when the implicit stiffness of the algorithm is chosen higher or equal to the real tangential stiffness of the structure K^{T} , hence $K^{T} \ge K^{T}$. This however is not valid anymore if the actuator dynamics are taken into account. Instability due to an overcompensation with the I-modification method will be illustrated in chapter seven.

2.3. Algorithms

There are multiple ways to integrate the differential equation of motion. All algorithms approximate the differential equation by finite difference equations and

can be evaluated in terms of certain characteristics, such as stability, energy conservation and numerical damping. The right choice of the algorithm depends on multiple factors, such as the system parameters, the test environment, the loading speed, the actuator delay and the desired accuracy. In general, choosing an algorithm will be a trade-off between stability, accuracy and computation time. The characteristics of the algorithm can be evaluated with a spectral decomposition.

2.3.1. Spectral decomposition

In algorithms, the dynamics (displacements, velocities and accelerations) of the new time step can be expressed in terms of the results of the last time step. For free vibration systems, the dependency of the new to the last time step can be expressed as in equation (2.4). A is called the amplification matrix and relates the dynamics of the new time step to the dynamics of the previous time step.

$$\begin{aligned} x_{n+1} &= A x_n \\ x_{n+1} &= A^{n+1} x_0 \end{aligned}$$
(2.4)

The Eigenvalues λ of the amplification matrix A determine the stability, the numerical frequency $\overline{\omega}$ and the numerical damping $\overline{\zeta}$ of the algorithm. They are not the same as the natural damping ζ and natural frequency ω of the structure model, but depend on them instead.

Depending on the algorithm, there are two complex conjugate eigenvalues and a third eigenvalue that is either zero or any real number. Any eigenvalue λ with real part *a* and imaginary part *b* can be expressed in polar coordinates. The numerical

damping $\overline{\zeta}$, the numerical frequency $\overline{\omega}$ and the damped numerical frequency $\overline{\omega}_D$ can be derived as shown in equation (2.5).

$$\lambda = a \pm ib = \sqrt{a^2 + b^2} e^{i \arctan(\frac{b}{a})} = e^{(-\overline{\zeta}\omega \pm i\overline{\omega}_D)h}$$
(2.5)

Generally, all the eigenvalues have to be checked and the most critical one is of final interest. If the solution is a bounded oscillatory response, two of the eigenvalues of Aare complex conjugates. The third eigenvalue is called the spurious root since it does not stand for a realistic numerical solution of free vibration [50]. The natural and the damped natural step will be expressed as $\Omega = \omega \Delta t = \omega h$ and $\Omega_D = \omega_D h$. The numerical step and damped numerical step are shown correspondingly as $\overline{\Omega} = \overline{\omega} h$ and $\overline{\Omega}_D = \overline{\omega}_D h$. From the shown relations the numerical damped step and the numerical damping result as shown in equation (2.6).

$$\overline{\Omega_{D}} = ArcTan(\frac{b}{a})$$

$$\overline{\zeta} = -\frac{\ln(a^{2} + b^{2})}{2\overline{\Omega}}$$
(2.6)

2.3.2. Stability

"A stable method is defined as one by which the numerical solution of a freevibration response will not grow without bound for any arbitrary initial conditions" [50]. This implies that the absolute values of all eigenvalues do not exceed the absolute value of *I*, i.e. all the eigenvalues have to be within the unit circle in the complex plane [51]. This can be seen more easily if the amplification matrix *A* is expressed in modal coordinates as $A = \phi^T \lambda \phi$ in the single degree of freedom (SDOF) case and $A = \Phi^T \Lambda \Phi$ in the multi degree of freedom (MDOF) case, where ϕ is the mode shape vector and Φ the mode shape matrix. For a SDOF equation (2.4) can be changed as follows:

$$x_{n+1} = A^{n+1} x_0 = \phi^T \lambda^{n+1} \phi x_0$$
(2.7)

For $|\lambda| > 1$ and a high number of n, λ^{n+1} and likewise x_{n+1} grow to infinity and the solution goes unstable. An algorithm is stable if a small change of the input leads to a small change in the response. It is unstable however, if the response grows to infinity. If the method is stable and the numerical solution approaches the exact solution for very small time steps, the method is convergent [50].

2.3.3. Shared characteristics of algorithms

Several algorithms will be presented in the following. The different algorithms differ in regard to stability, numerical damping and period distortion. Generally, instability can be seen in two different ways. One is, that at least one eigenvalue of the amplification matrix exceeds the absolute value of I. Simultaneously, this instability can be seen as negative numerical damping. This means that energy is added into the system and instability may happen. While most implicit methods are unconditionally stable, the explicit methods are conditionally stable, depending on the step Ω . The natural step Ω also affects the period distortion. While the natural and numerical frequencies are equivalent for infinitely small time steps, the period distortion generally grows with increasing time step.

2.3.4. Newmark method

The first group of algorithms to find widespread use was the Newmark family, in which approximate relations are used to express the displacement and velocity vectors u_{n+1} and \dot{u}_{n+1} at the new time t_{n+1} in terms of the new acceleration vector \ddot{u}_{n+1} and current values of displacements, velocities and accelerations.

$$\dot{u}_{n+1} = \dot{u}_n + (1 - \gamma)\ddot{u}_n h + \gamma \ddot{u}_{n+1} h$$

$$u_{n+1} = u_n + \dot{u}_n h + (\frac{1}{2} - \beta)\ddot{u}_n h^2 + \beta \ddot{u}_{n+1} h^2$$

$$M\ddot{u}_{n+1} + (1 + \alpha)C\dot{u}_{n+1} - \alpha C\dot{u}_n + (1 + \alpha)Ku_{n+1} - \alpha Ku_n = (1 + \alpha)P_{n+1} - \alpha P_n$$
(2.8)

Equation (2.8) shows the Newmark α method or Hilber-Hughes-Taylor method. The elimination of u_{n+1} and \dot{u}_{n+1} from the equation of motion at time t_{n+1} results in a system of equations for the acceleration vector at t_{n+1} . *h* denotes the integration time step. The Newmark algorithm is popular due to its simplicity, however it has shortcomings relating to high frequency components, algebraic constraints and lack of exact conservation properties [24].



Figure 2.5: Newmark methods

Figure 2.5 shows the Newmark family methods. The tuning of the parameters α , β and γ creates both explicit and implicit methods with different characteristics. Explicit methods compute the response of the structure of step *i*+*1* based on the results of step *i*. They are easier to implement and usually preferred for hybrid simulations, however usually have more restrictive stability criteria related to the natural step Ω . Implicit methods require information about the structural response at the displacement target in order to satisfy equilibrium at the end of the step. They provide for better stability characteristics and enable the use of bigger time steps.

The operator-partitioning algorithm is a combined implicit-explicit integration algorithm. The following predictors are calculated explicitly.

$$\hat{u}_{n+1} = \dot{u}_n + (1 - \gamma)\ddot{u}_n h$$

$$\hat{u}_{n+1} = u_n + \dot{u}_n h + (\frac{1}{2} - \beta)\ddot{u}_n h^2$$
(2.9)

The predicted displacements are applied to the physical model and the force response is measured. Then one can solve for the new acceleration and then update with correctors.

$$\begin{aligned} \ddot{u}_{n+1} &= \tilde{M}^{-1} \tilde{P}_{n+1} \\ \dot{u}_{n+1} &= \hat{u}_{n+1} + \gamma \ddot{u}_{n+1} h \\ u_{n+1} &= \hat{u}_{n+1} + \beta \ddot{u}_{n+1} h^2 \end{aligned}$$
(2.10)

where $\tilde{M} = M + h\gamma C + \beta h^2 K$ and $\tilde{P}_{n+1} = P_{n+1} - C\hat{u}_{n+1} - K\hat{u}_{n+1}$. The advantage of using the operator splitting method is that unconditional stability is guaranteed for nonlinear structures of the softening type. In this study the OS-method is applied with only one iteration step assuming a linear structure stiffness. Combescure and Pegon [10] used a linearization of the earlier shown α -method, the α -Operator Splitting. By using a predictor step, this implicit scheme becomes non-iterative. The feedback force is approximated by

$$r_{n+1}(d_{n+1}) \approx K^{I} d_{n+1} + (\tilde{r}_{n+1}(\tilde{d}_{n+1}) - K^{I} \tilde{d}_{n+1})$$
(2.11)

where \tilde{d}_{n+1} is the predicted displacement and K^{I} the initial stiffness. I.e. they applied the I-modification scheme within the first and only iteration step.

2.4. Summary

This chapter explained the different components of hybrid testing and all currently used algorithms, prediction schemes, inter- and extrapolation schemes, as well as error compensation schemes. Experimental errors are the most significant error source in hybrid testing and results to a large degree from bad actuator tracking. Compensation schemes such as numerical dissipation or the I-modification method cannot improve the actuator tracking but only mitigate its harmful effect on the hybrid system. The next chapter illustrates the reasons for bad actuator tracking and explains why good tracking requires the integration of the actuator-structureinteraction into the control design.

3. ACTUATOR DYNAMICS AND STABILITY

This chapter shows in what way the actuator-structure-interaction and specifically the structure stiffness affect the tracking of both hydraulic and electromagnetic actuators. The transfer functions of both types of actuators are derived and their similarities shown. The stability is analyzed with the poles of the transfer functions in the frequency domain.

3.1. Stability theory of linear systems

Creating a transfer function requires the transformation of the differential equation of the system from the time domain into the frequency domain, also called "s-domain", by using the Laplace transform. The transfer function can then be expressed as the relation between the input and output.

$$\frac{x(t)}{r(t)} = \frac{output}{input} = H(s) = \frac{X(s)}{R(s)}$$
(3.1)

where r is the reference input, x the reference output and H the transfer function relating the output to the input.

3.1.1. Poles of the transfer function

The solutions of the denominator of the transfer function are called the poles of the system. The location of the poles in the s-domain gives information about the system dynamics in regard to stability, damping and oscillatory behavior. Equation (3.2)

shows that the denominator consists of real poles and pairs of complex conjugate poles.

$$H(s) = \frac{output}{input} = \frac{X(s)}{R(s)} = \frac{1}{s+p_1} + \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} + \dots$$
(3.2)

The time response for a real pole is

$$H(s) = \frac{1}{s + \sigma} \to h(t) = e^{-\sigma t}$$
(3.3)

The time response of a complex conjugate pole is an oscillating response.

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \to h(t) = \frac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\sigma t} (\sin \omega_d t)$$
(3.4)

It is obvious that a response is only stable if all poles have a negative real part, so that $e^{-\sigma t} \rightarrow 0$ for $t \rightarrow \infty$. In other words, stability is only achieved if all poles are placed in the left half plane (LHP).

3.2. Similarities between both actuators

The following paragraphs explain and compare the functionalities of the hydraulic and electromagnetic actuator. The dynamics of both actuators are remarkably similar. For example, a "natural velocity feedback" is observed in the hydraulic actuator dynamics (see for example [11]). The back EMF plays an identical role in the electromagnetic actuator dynamics. The linearization of the electromagnetic actuator dynamics has a real pole and a complex conjugate pair of poles. The hydraulic actuator has three poles with identical roles, and additional poles depending on the modeling of the servovalve dynamics [21, 32]. The analysis and strategies presented in this study apply to both actuators.

3.3. Hydraulic actuator

During a test involving servo-hydraulic actuation, a servovalve controller compares a command signal to a feedback signal and sends the generated valve command signal to the servovalve to drive the valve spool. The spool controls the hydraulic fluid into the chambers of the actuator. The pressure difference between the two chambers multiplied by the actuator piston area produces the force applied to the test structure [60].

3.3.1. Valve command signal

The valve command signal *v* can be expressed as

$$v = C_F[K_n(r-x) + K_d(\dot{r} - \dot{x})]$$
(3.5)

where *r* is the command signal, *x* is the feedback signal and C_F converts these physical signals into voltage signals. K_p and K_d are respectively the proportional and derivative gain settings in the servovalve controller.

3.3.2. Dynamics of servovalve

The dynamics for a three-stage servovalve can be described as

$$v = m_{ev} \ddot{x}_{v} + c_{ev} \dot{x}_{v} + k_{ev} x_{v}$$
(3.6)
where x_v is the main-stage spool opening and m_{ev}, c_{ev}, k_{ev} are the equivalent mass, damping and stiffness of the servovalve.

3.3.3. Flow property of the servovalve

The flow property of the servovalve, which relates the main-stage spool opening to the hydraulic flow it controls, can be formulated as

$$Q = K_{v} x_{v} \sqrt{1 - \frac{x_{v}}{|x_{v}|} \frac{p}{p_{s}}}$$
(3.7)

where Q is the flow into the actuator chambers, K_{ν} is the main-stage servovalve flow gain, p_s is the hydraulic supply pressure and p is the load pressure. The square root term is named "load pressure influence" and introduces nonlinearity to the system.

3.3.4. Conservation of mass

The actuator dynamics are given by

$$Q = K_a \dot{p} + C_1 p + A \dot{x} \tag{3.8}$$

where K_a is the hydraulic fluid compressibility coefficient, C_1 is the leakage coefficient, A is the actuator piston area and \dot{x} is the piston velocity. The conservation of mass shows a feedback path from the structural velocity response \dot{x} to the hydraulic flow into the actuator. This resulting loop is referred to as the "natural velocity feedback".

3.3.5. Linearization

The presented actuator dynamics show a relationship between the oil flow in the piston and the load pressure. The interaction force changes the pressure difference in the piston which can inhibit or increase the oil flow.



Figure 3.1: Pressure difference in actuator

Static equilibrium in Figure 3.1 yields

$$p = p_A - p_B = \frac{F}{A} \tag{3.9}.$$

Substituting (3.9) into (3.7) shows that the oil flow depends on the interaction force:

$$Q = K_v x_v \sqrt{1 - \frac{x_v}{|x_v|} \frac{F}{Ap_s}}$$

(3.10)

The piston velocity then yields

$$\dot{x} = K_v \frac{x_v}{A} \sqrt{1 - \frac{x_v}{|x_v|} \frac{F}{Ap_s}} - K_a \frac{\dot{F}}{A^2} - C_1 \frac{F}{A^2}$$
(3.11)

This shows that the piston movement is inhibited by the actuator-structureinteraction force. Like stated in the introduction this means for a displacement controlled actuator, that the applied displacement x depends on two input variables: the reference command displacement u and the interaction force F.



Figure 3.2: Inputs for displacement controlled actuator

Figure 3.2 repeats the earlier shown diagram. The two transfer functions of the system both yield displacement output but have different input parameters. $H_{ux}(s)$ gives the resulting displacement output due to the desired displacement input, while $H_{fx}(s)$ gives the resulting displacement output due to the interaction force. For a linear system, those two displacement outputs superimposed form the final actuator displacement. The transfer functions result from the actuator properties, such as piston area, actuator mass, oil bulk modulus, supply pressure, oil density, oil viscosity and others. The two transfer function in equations (3.12) and (3.13) result from the linearization of the actuator dynamics according to Kuehn et al [25].

$$H_{ux}(s) = \frac{K_{actgain}}{s(s+p)(s^2 + 2\zeta\omega_a s + \omega_a^2)}$$

(3.12)

$$H_{fx}(s) = \frac{s + 2\zeta\omega_a}{s(s^2 + 2\zeta\omega_a s + \omega_a^2)}$$
(3.13)

 ω_a denotes the eigenfrequency of the actuator oil column, ζ the inherent actuator damping and *p* the reciprocal of the inherent actuator delay.

3.3.6. Effect of large structure stiffness

For a linear structure stiffness without any mass or damping, the interaction force can be calculated directly from the applied actuator displacement. The open loop transfer function H_{ol} in equation (3.14), relating the displacement output to the input signal, represents the structure stiffness as a parameter which determines the system dynamics.



Figure 3.3: Block diagram of open loop transfer function

$$H_{ol} = \frac{x}{u} = \frac{H_{ux}}{1 + kH_{fx}}$$

$$= \frac{K_{actgain}}{s^4 + (p + 2\zeta\omega_a)s^3 + (k + 2p\zeta\omega_a + \omega_a^2)s^2 + (k(p + 2\zeta\omega_a) + p\omega_a^2)s + 2\zeta\omega_a pk}$$
(3.14)

For proportional feedback control, the input voltage u is the displacement error, hence the difference between reference displacement r and applied displacement x, multiplied with a proportional gain K_p , $u = K_p(r-x)$. The closed loop transfer function then relates the reference command r to the applied displacement x.

$$H_{cl} = \frac{x}{r} = \frac{K_{p}H_{ux}}{1 + kH_{fx} + K_{p}H_{ux}} = \frac{K_{p}K_{actgain}}{s^{4} + (p + 2\zeta\omega_{a})s^{3} + (k + 2p\zeta\omega_{a} + \omega_{a}^{2})s^{2} + (k(p + 2\zeta\omega_{a}) + p\omega_{a}^{2})s + K_{p}K_{actgain} + 2\zeta\omega_{a}pk}$$
(3.15)

The stability of the system depends on all parameters and will be analyzed with Routh criteria as well as bode and root locus plots [14]. Routh criteria reveals an upper stability limit for the proportional gain:

$$K_{p} < \frac{2\zeta\omega_{a}^{3}(k(p+2\zeta\omega_{a})+p(p^{2}+2p\zeta\omega_{a}+\omega_{a}^{2}))}{K_{actgain}(p+2\zeta\omega_{a})^{2}}$$

$$= \frac{2\zeta\omega_{a}^{3}p(p^{2}+2p\zeta\omega_{a}+\omega_{a}^{2}))}{K_{actgain}(p+2\zeta\omega_{a})^{2}} + \frac{2\zeta\omega_{a}^{3}}{K_{actgain}(p+2\zeta\omega_{a})}k$$
(3.16)

Equation (3.16) illustrates that the upper limit for the proportional gain grows with increasing stiffness. This however, does not justify the application of a higher gain for higher stiffness, as will show the following root locus plot. As the influence of the structure stiffness and the controller gains on the system both depend on the oil column frequency ω_a , the closed loop transfer function is now represented in non-dimensional parameters.

$$H_{cl} = \frac{Gain}{S^{4} + (P + 2\zeta)S^{3} + (K_{ratio} + 2P\zeta + 1)S^{2} + (K_{ratio}(P + 2\zeta) + P)S + Gain + 2\zeta PK_{ratio}}$$
(3.17)

where

$$S = \frac{s}{\omega_a} \quad P = \frac{p}{\omega_a} \quad K_{ratio} = \frac{k_s}{k_a} = \frac{\omega_s^2}{\omega_a^2} \quad Gain = \frac{K_p K_{actgain}}{\omega_a^4}$$
(3.18)

The stiffness ratio K_{ratio} represents the relations between the spring stiffness and the actuator stiffness. Figure 3.4 plots the root locus or the closed loop transfer function and changing gain for P = 100 and $\zeta = 0.1$.



Figure 3.4: Root locus for hydraulic actuator and different stiffness ratios

Figure 3.4 shows that for a growing stiffness ratio the poles shift upwards and towards the imaginary axis. Hence, this movement is independent of the oil column frequency. This effect is likewise present for the electromagnetic actuator.

3.4. Electromagnetic actuator

The use of an electromagnetic actuator will allow for a smaller and cleaner test setup where no hydraulic oil pressure is necessary. Further advantages of electrodynamic shakers are the long stroke, good linearity and a wide frequency response. The structure of an electrodynamic shaker resembles to a common loudspeaker, but is more robust. A coil of wire is suspended in a fixed radial magnetic field. When a current is passed through this coil, an axial force is produced in proportion to the current.

The electrical impedance increases with frequency due to the skin effect. When the coil moves within the magnetic field, a voltage is generated across the coil in

proportion to the velocity. This back EMF (electromotive force) is seen in the electrical domain as an increase of the coil impedance and reflects the mechanical activity in the electrical circuit.

The performance is generally limited by different factors, such as the thermally determined maximum coil current, the maximum current capacity of the amplifier, the maximum voltage capability of the amplifier, the stroke capability of the shaker or the maximum armature force capability.

Figure 3.5 shows the schematic of an electromagnetic actuator. A wire coil moves within a fixed magnetic field and represents the accelerating element in the actuator.



Figure 3.5: Electromagnetic actuator

A conductor carrying a current in a magnetic field experiences a force perpendicular to both the magnetic field lines and the direction of the current flow. In the case of the actuator, a current *i* in the coil results in a force *Bli*, where *B* is the magnetic field and *l* is the conductor making up the coil [26]. The motion of the coil is resisted by inherent mechanical damping in the actuator as well as by eddy-current damping. The effects of these may be lumped approximately into a linear viscous damping coefficient, *c*. The equation of motion of the coil is therefore $m\ddot{x} + c\dot{x} = Bli - F$, where *m* is the moving mass, *x* is its displacement, and *F* is the force at the interface with the tested structure. The motion of the wire coil in the magnetic field in turn results in a voltage $Bl\dot{x}$, called the back EMF, that opposes the externally applied voltage. If the inductance and resistance of the coil are *L* and *R* respectively, then the control input voltage *u* and the current *i* in the coil are related by $u - Bl\dot{x} = L\frac{di}{dt} + Ri$.

The above two equations can be represented in state space form as

$$\begin{bmatrix} Bl & 0 & L \\ c & m & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -R \\ 0 & 0 & Bl \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} F$$

$$\begin{cases} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{C}{m} & \frac{Bl}{m} \\ 0 & -\frac{Bl}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} u + \begin{bmatrix} 0 \\ -\frac{1}{m} \\ 0 \end{bmatrix} F$$
(3.19)

where $x_1 = x$, $x_2 = \dot{x}$ and $x_3 = i$ represent the displacement, velocity and current.

3.4.1. Linearization

Figure 3.6 shows the open loop block diagram for this system where the force coupling is modeled as the product of the applied actuator displacement and the structure stiffness. The transfer function from the control input u to the actuator displacement x can be derived as

$$H_{ol} = \frac{x}{u} = \frac{Bl}{mLs^3 + (mR + cL)s^2 + (kL + cR + (Bl)^2)s + kR}$$
(3.20)

It can be seen that again the poles of the transfer function are determined not only by the actuator properties, but also by the structure stiffness.



Figure 3.6: Open loop block diagram for electromagnetic actuator

3.4.2. Effect of large structure stiffness

Like before for the hydraulic actuator, the closed loop transfer function is derived for proportional feedback control (equation (3.21)) and the maximum allowable gain is derived (equation (3.22)) using again the Routh criterion [14].

$$H_{cl} = \frac{x}{r} = \frac{K_{p}Bl}{mLs^{3} + (mR + cL)s^{2} + (kL + cR + (Bl)^{2})s + kR + K_{p}Bl}$$
(3.21)
$$K_{p\max} \le \frac{((Bl)^{2} + cR)(cL + mR) + ckL}{Bl \ Lm}$$
(3.22)

Again the maximum allowable gain is a function of the structure stiffness and the transfer function can be represented in nondimensional parameters. The actuator frequency ω_a and damping ζ_a have been derived for free motion as

$$\omega_a = \sqrt{\frac{(Bl)^2 + cR}{Lm}}$$
 and $\zeta_a = \frac{cL + Rm}{\sqrt{4Lm((Bl)^2 + cR)}}$ from the two complex conjugate pair of

poles of the transfer function in equation (3.20), such that

$$H_{ol}(k=0) = \frac{1}{s} \frac{Bl}{mLs^2 + (mR+cL)s + cR + (Bl)^2} = \frac{1}{s} \frac{Bl/mL}{s^2 + 2\zeta\omega_a s + \omega_a^2}$$
(3.23)

With the substitutions
$$R_n = \frac{R}{L\omega_a}$$
, $S = \frac{s}{\omega_a}$, $G_a = \frac{Bl}{Lm\omega_a^3}$ and

 $K_{ratio} = \frac{k}{k_a} = \frac{k}{m\omega_a^2} = \frac{kL}{(Bl)^2 + cR}$ the open loop transfer function yields

$$H_{ol} = \frac{G_a}{S^3 + 2\zeta_a S^2 + (1 + K_{ratio})S + R_n K_{ratio}}$$
(3.24)

The stiffness ratio K_{ratio} again relates the stiffness of the structure to the "stiffness" of the actuator in free motion. Figure 3.7 shows the root locus plot for the nondimensional transfer function and increasing gain. The used parameters correspond to the actuator which is used in the later examples.



Figure 3.7: Stiffness dependent root locus of closed loop transfer function

It can be seen that for given proportional gain and increasing stiffness ratio, the real pole shifts farther into the left half plane, while the complex conjugate pair of poles converges towards the imaginary axis. The real pole shift leads to faster dynamics, and so does not affect the response significantly. The complex conjugate pair of poles on the other hand leads to oscillatory response. The settling time and damping ratio of the system are related to the real part of these poles. This implies that for a high structure stiffness, the system has a very low damping and high settling time. Oscillations will not be effectively damped out and in the worst case, system uncertainties can shift the poles into the right half plane, resulting in instability. The following example shows how an uncertainty in the form of a small time delay can easily destabilize a system with high structure stiffness.

3.5. Instability due to high stiffness

A closed-loop system is said to be robustly stable if the controller stabilizes a class of perturbations of the nominal system. If the open loop transfer function H_{ol} is perturbed by time delay τ , then the perturbed transfer function is given by

$$H_{\tau} = e^{-s\tau} H_{ol} \approx \frac{1}{1 + \tau s} H_{ol}$$
(3.25)

The class of such transfer functions may be considered as multiplicative perturbations of the form $H_{\tau} = H_{ol}(1+W(s)\Delta(s))$, where W(s) is a real function representing the magnitude of the perturbations and $\Delta(s)$ is such that $|\Delta(s)| \le 1$ and represents phase uncertainty [14]. For the case of equation (3.25), W(s) is given by

$$W(s) = \left| \frac{H_{\tau} - H_{ol}}{H_{ol}} \right| \approx \left| \frac{-\tau s}{1 + \tau s} \right|$$
(3.26)

Robust stability is met if $|H_{cl}(j\omega)W(j\omega)| < 1$, where H_{cl} is the earlier derived closed loop transfer function of the original system without delay [14]. For the presented electromagnetic actuator, this requires

$$\left| H_{cl}(j\omega)W(j\omega) \right| = \frac{K_{p}Bl}{\sqrt{\left[mL\omega^{3} - (kL + Bl^{2} + cR)\omega\right]^{2} + \left[K_{p}Bl + kR - (mR + cL)\omega^{2}\right]^{2}}} \frac{\omega\tau}{\sqrt{1 + (\omega\tau)^{2}}} < 1$$
(3.27)

for all ω . Considering the values Bl = 26.31 N/A, $R = 3.58\Omega$, L = 0.08 H, c = 11 Ns/m and m = 2.275 kg, corresponding to the actuator used in this study, and choosing a proportional gain $K_p = 0.8 K_{pmax}$, which is required in order to get good actuator tracking, the frequency function in equation (3.27) is plotted in Figure 3.8 for a time delay of $\tau = 2 ms$. Two stiffness ratios are considered, $K_{ratio} = 0$, i.e. free

motion, and $K_{ratio} = 3$ ($k = 30 \frac{kN}{m}$).



Figure 3.8: Robust stability criterion fails due to high stiffness

Figure 3.8 shows that the system is robustly stable in free motion for perturbations in form of time delays up to 2 *ms*. When the system is attached to a structure of very high stiffness however, then it lacks robust stability. Summarizing, a high structure stiffness leads to low system damping resulting in high overshoot and highly oscillatory time response. Furthermore, system uncertainties can quickly lead to instability. It is thus important that the actuator control adjusts to the structure. This can be achieved by controlling the actuator impedance, as will be explained in the following chapter.

4. IMPEDANCE CONTROL

It is evident from the previous discussion, that it is important to explicitly account for actuator-structure interaction in the control design of a test system. As outlined in the introduction, a strategy to do so is using feedforward. Here, this strategy is further motivated using the notion of impedance control [17].



Figure 4.1: External force on coupled system

Figure 4.1 shows an additional external force acting on the coupled actuatorstructure system. The closed loop transfer function of this system can be written as

$$x = H_{cl}r - \frac{R + Ls}{Bl K_p} H_{cl} F_{ext}$$
(4.1)

The second transfer function of equation (4.1) can be interpreted as the *compliance* (frequency-dependent flexibility) of the combined actuator-structure system. The inverse of this is the *impedance* (frequency-dependent stiffness). Clearly, if the impedance is large, then the response of the system is insensitive to force disturbances at the interface, and hence to uncertainties in the structure model. So, the design of the test system with explicit consideration of actuator-structure

interaction may be viewed as designing a suitable impedance of the system. The impedance however cannot be arbitrarily increased, because this would imply shifting the poles of the closed-loop system in a manner that would result in undesirable dynamic behavior, and in the worst case, instability. The *target impedance* is therefore established by the desired locations of the closed-loop poles, which in turn are based on well-known heuristics for closed-loop pole placement [14].

From equation (4.1), the impedance \hat{K} of the coupled system can be considered as the sum of the actuator impedance and the structure stiffness, as shown in equation (4.2).

$$\hat{K} = \left| \frac{x_{ext}}{F_{ext}} \right| = \frac{mLs^3 + (mR + cL)s^2 + (cL + (Bl)^2)s + BlK_p}{R + Ls} + k$$
(4.2)

Given a target impedance, it is therefore seen, that the actuator impedance and the structure stiffness must complement each other, i.e., the actuator impedance has to decrease if the structure stiffness increases and vice versa. Furthermore, equation (4.2) can be written as

$$\hat{K} = \frac{Lms^{2} + (mR + cL)s + (cR + Bl^{2})}{R + Ls}s + \frac{BlK_{p}}{R + Ls} + k$$
(4.3)

from which it can be seen that with a proportional feedback controller, the gain is the only tool that allows modification of the actuator impedance in accordance to the structure stiffness. Due to the finite actuator impedance, the interaction force F affects the actuator movement. The actuator displacement in free motion is superimposed by the force dependent displacement x_F .

$$x_{F} = \frac{R + Ls}{Lms^{3} + (mR + cL)s^{2} + (cR + Bl^{2})s + BlK_{p}}F$$

$$x_{F}(s = 0) = \frac{R}{BlK_{p}}F$$
(4.4)

Note, that there is even statically the actuator is deformed by the interaction force. The actuator must allow this deformation in order to maintain the required impedance. On the other hand, accurate actuator tracking is desired. Feedback solutions, such as a PID controller may improve tracking in particular cases, but the objective here is to adjust the system impedance and decouple the structure from the actuator. Feedforward will now be presented as a way to compensate for the deformation due to the interaction force while maintaining the target actuator impedance.

4.1. Impedance control by feedforward

Feedforward applies another input to the actuator which helps improve its tracking while maintaining the impedance of the coupled system. In this work, the feedforward signal is designed to compensate for the static deformation only. This means that for a constant reference displacement, feedforward eliminates the effect of the actuator-structure-interaction. It is practically impossible to completely compensate for the interaction in the dynamic case, as this would require the prediction of the interaction force at the moment the feedforward signal must be applied. The closed loop system is again a two input system, one input being the reference displacement command r and the other one still the interaction force F. (find derivation in appendix C).

$$x = \frac{H_{ux}K_p}{1 + H_{ux}K_p} \left(r - \frac{R + Ls}{BlK_p}F\right)$$
(4.5)

In order to compensate for the interaction force, the required feedforward is $ff = \frac{R + Ls}{Bl K_p} F$. But this transfer function is improper. So the feedforward term is

approximated with the static component $ff = \frac{R}{Bl K_p} F$. Chapter six will present

different feedforward models and explain their advantages and disadvantages.

4.2. Summary

So far it has been explained that the actuator must allow a deformation in response to the interaction force. This way the actuator has a certain impedance, which is required to adjust to a changing structure stiffness. As the poles of the system compliance or the zeros of the impedance are equivalent to the poles of the closed loop transfer function, the system dynamics can be controlled by controlling the impedance. A target impedance can be defined from design heuristics which relate the complex conjugate pair of poles to the system response in the time domain. Feedforward is as a tool to compensate for the actuator-structure-interaction while maintaining the requirement of the actuator flexibility. A stiffer structure requires a softer actuator, which means that the actuator deforms more to the interaction force but compensates for it with a higher feedforward input.

5. SYSTEM SETUP AND IDENTIFICATION

In chapter 3, the transfer function of an electromagnetic actuator has been derived. This chapter derives all unknown actuator parameters by comparing different measured frequency response functions (FRF) to the simulated FRF from the assumed actuator model. The good match between the measurement and simulation shows that the actuator model is valid, that the system is fully understood, and that the derived parameters are sufficiently accurate. Moreover this chapter presents the whole test setup, including the host and target computer, amplifier, actuator, test specimen, as well as all measurement devices.

5.1. System Setup



Figure 5.1: System deployment diagram

Figure 5.2 shows the test setup including the structure stiffness. The actuator is mounted on a steel plate which lies on the shown table. Different measurement devices at the end of the actuator arm are used to document the applied displacement (LVDT), acceleration (accelerometer) and interaction force (load cell). The LVDT is placed opposite of the actuator to avoid an interaction between the magnetic field of

the actuator and the wire coil inside the LVDT. The different components will now be explained in detail.



Figure 5.2: Test setup

5.1.1. Host computer and real-time processor

The setup shows two computers, the host and target machine. If the test is run in non-real-time, then the target computer can likewise be used as the host computer. For real-time simulations the host is needed, running in non-real-time, downloading the executable file to the target and serving as the user interface, while the target machine performs all computations in real-time. The target machine can be booted into labview windows or labview real-time.

5.1.2. Amplifier

The amplifier receives a command from the processor and sends the amplified command in form of a current to the actuator. The amplifier can be run in either voltage mode or current mode. In voltage mode the current output depends on the shaker impedance and the mechanical load impedance. Due to the back EMF and the coil inductance the actuator movement is damped and produces an approximate "constant velocity response". This mode is therefore used for most tests.

The current, which the amplifier sends to the actuator and which flows through the actuator coil, can be read from the current monitor output. In current mode this current monitor output represents the input signal multiplied by the amplifier gain in current mode. In voltage mode the current monitor output will not be proportional anymore to the input signal, as the current flow will depend on the actuator impedance.

In current mode the shaker has minimum effect on the system damping. The output current is directly related to the input voltage, regardless of the shaker impedance or load impedance, so that the voltage input is directly related to the applied force. The current mode is later used to derive parameters and properties of the actuator.

5.1.3. Measurement devices

Apart from the voltage input and the current flow in the actuator coil, there are three more dimensions which can be measured: the actuator displacement, the actuator acceleration and the interaction force with the connected structure. The used devices are described briefly.

5.1.3.1. LVDT

The actuator displacement is measured with an LVDT and the corresponding LVDT Signal Conditioner (LVC 2412). It provides a low distortion sine wave to excite the LVDT and employs a synchronous demodulator to convert the LVDT's AC output signal to more useful DC outputs proportional to core positions. Additional circuitry provides span and zero adjustability and a two-pole low pass output filter. No phase adjustment is needed and a low noise system response is achieved.

5.1.3.2. Load cell

The used load cell (model LC202-500) can take loads up to 500 lbs. Testing low stiffness, this high range load cell unfortunately has a relatively noisy response. The AC powered signal conditioner (model DMD-465WB) features a frequency response up to $2 \ kHz$. It contains a precision differential instrumentation amplifier with filtered output and a highly regulated, low noise, adjustable output bridge excitation source.

5.1.3.3. Accelerometer

The model ADXL203 is a high precision, low power, dual axis accelerometer with signal conditioned voltage outputs, measuring acceleration with a full-scale range of $\pm 1.7g$. The typical noise floor is $110\mu g/\sqrt{Hz}$. For the low frequency range the LVDT signal therefore has been taken as a more accurate to derive the transfer function. For the chosen excitation frequency of 5V the output yields 1000mV/g.

5.2. System identification

Some properties of the amplifier and actuator are known and have already been shown. In the following, the missing parameters will be derived experimentally. Those parameters are for example the amplifier gain, the eigenfrequencies of the system in current and voltage mode, the inductance of the coil and the eddy current damping, its mechanical damping, as well as the stiffness of the four rubber bands which are inherent in the actuator.

Different frequency response functions are measured, using as the in- and output the actuator displacement (LVDT), the actuator acceleration (accelerometer), the current flow in the actuator coil (current monitor output of the amplifier) and the stimulus (input signal to the amplifier in either volts for voltage mode or amperes in current mode). The comparison of the measurements with the modeled responses allows the derivation of the missing parameters and shows that the linearized actuator model is accurate enough.

5.2.1. Current mode

If the system is run in current mode, then the sent signal from the processor directly relates to the current in the actuator coil.

5.2.1.1. Decay method

Running the system in the CURRENT OFF mode reduces the system damping to the mechanical damping only. The amplifier basically absorbs the eddy current damping and the back EMF, which is created in the coil due to its movement within the magnetic field. This way no electrical damping is present. The actuator is released from a position outside of equilibrium and the displacement is measured via the LVDT. Figure 5.3 shows one of the test samples. Due to the mechanical damping of the actuator, the actuator returns to the neutral position after a few oscillations.



Figure 5.3: Decay method in CURRENT OFF mode

The mechanical damping ratio ζ_m is derived with the decay method, represented in equation (5.1), where u_i is the maximum displacement in cycle *i* and u_{i+n} the maximal displacement *n* cycles later.

$$\zeta_m = \frac{1}{2\pi n} \ln(\frac{u_i}{u_{i+n}}) \tag{5.1}$$

The average mechanical damping ratio results as $\zeta_m \approx 0.15$.

5.2.1.2. TF displacement/current

In current mode, the resonance frequency of the system is very low. Therefore the frequency response in current mode is measured with an LVDT providing that the accelerometer is noisy for the corresponding low frequencies. By testing the system in current mode, the high source resistance of the amplifier provides very little damping. The high resonance peak in the response is limited by the mechanical damping of the actuator, hence the internal damping in the silicone rubber and rolling friction of the bearings. As in current mode the effect of the back EMF and

the coil impedance is compensated for, the relation between the current input to the amplifier and the resulting actuator displacement, which is measured with the LVDT can be represented by the following block diagram.



Figure 5.4: Current-displacement block diagram

It can be seen that the current input is directly proportional to the resulting force. If there were no rubber bands and no mechanical damping in the actuator, the input current would also be proportional to the acceleration. The open loop transfer function of the system relating the actuator displacement to the current input can now be written as:

$$H_{ix} = \frac{BlG_c}{ms^2 + c_w s + k} \tag{5.2}$$

Again it should be noticed that the damping c in the block diagram is only due to the mechanical damping c_m and not electrical damping, it is therefore represented as c_m in equation (5.2). The actuator has four internal rubber bands, acting like a low structure stiffness connected to the actuator. The main purpose of those rubber bands is, that the actuator can get centered and that no over-travel in open loop occurs. This stiffness k represents the sum of the stiffness of the structure and the rubber bands. The two figures below show the measured frequency response function (FRF) for magnitude and phase.



Figure 5.5: Magnitude plot in current model



Figure 5.6: Phase plot in current mode

This system has one pair of complex conjugate poles, which cause a resonant peak. The system damping and the stiffness of the rubber bands can therefore be derived from the resonance peak, which is found at about $f_{current}=1.53Hz=9.6rad/s$. Knowing the moving mass of the actuator, the total stiffness due to the four diagonal rubber bands approximates

$$k_{rubber} = m\omega_{current}^2 \approx 210 \frac{N}{m} = 1.2 \frac{lb}{in}$$
(5.3)

From the resonance peak, the damping ratio can be approximated as shown in Figure 5.5, resulting in $\zeta_m = 0.5(\beta_1 - \beta_2) \approx 0.2$. This is a confirmation of the earlier received damping ratio in equation (5.1). As the resonance of this system is known, the absolute mechanical damping of the actuator can be derived.

$$\frac{c_m}{m} = 2\zeta_m \omega_{current} = 4\pi \zeta_m f_{current}$$

$$c_m = 4\pi m f_{current} \zeta_m \approx 43.7 \frac{Ns}{m} \zeta_m = 6 \rightarrow 9 \frac{kg}{s} = 0.03 \rightarrow 0.05 \frac{lb s}{in}$$
(5.4)

The mechanical properties of the actuator have been derived. The remaining unknown parameter in equation (5.2) is the amplifier gain in current mode G_c . In current mode, the input signal to the amplifier is read as the desired current in amperes. Different measurements relating the input current to the applied displacement allow for the derivation of the amplifier gain, resulting in an average value of $G_c \approx 4.25$. This means that in current mode the stimulus, which is a current in amperes, leads to an actuator displacement as follows:

$$d[in] = \frac{Bl}{k_s} G_c \ input[A] = 4.93 \frac{in}{A} G_c \ input[A] \approx 21 \ input[A] \tag{5.5}$$

Further tests are presented which confirm the derived parameters.



5.2.1.3. TF current monitor output / current input

Figure 5.7: Amplifier gain in current mode

The current monitor output documents the current output of the amplifier which is equivalent with the current flowing through the actuator coil. Figure 5.7 shows the current monitor output over the applied current stimulus. The peak occurs at the earlier shown actuator eigenfrequency in current mode. This is reasonable as at this eigenfrequency the actuator has the highest velocity. The amplifier has to compensate for the resulting high back EMF by providing a higher current. As the velocity increases linearly with growing frequency it is reasonable that the amplifier gain likewise increases linearly. This effect is comparable to the earlier shown effect in the hydraulic actuator, which was unable to apply force at the resonance frequency, as the movement of the actuator created a vacuum in the oil column.

5.2.1.4. Confirmation of the derived parameters

The measured frequency response function relating the actuator displacement to the current input is tried to be matched with the derived values. The derived values result

as accurate, although a slightly higher amplifier gain and slightly lower mechanical damping seem more convenient to match the results. In the following the used parameter values are used to match the measured FRF.

$$m = 2.275 \, kg \quad c = c_m = 5 \frac{Ns}{m} \quad k = 210 \frac{N}{m} \quad Bl = 26.31 \frac{N}{A} \quad G_c = 4.25 \quad (5.6)$$

Equation (5.7) shows the frequency dependent magnitude and phase of the actuator displacement as a function of the current flow. Figure 5.8 compares the measured FRF with the modeled FRF using the parameters in equation (5.6).

$$H_{ix,ol}(s) = \frac{BlG_c}{ms^2 + c_m s + k} \to H_{ix,ol}(j\omega) = \frac{BlG_c}{k - m\omega^2 + jc_m\omega}$$

$$Mag_{ix,ol}(\omega) = \frac{BlG_c}{\sqrt{(k - m\omega^2)^2 + (c_m\omega)^2}} \quad \theta_{ix,ol}(\omega) = arcTan[\frac{c_m\omega}{k - m\omega^2}]$$
(5.7)



Figure 5.8: Comparison of measured and modeled FRF in current mode

The measured phase response again shows a continuously growing phase lag over 2 Hz. The theoretical model has only two poles, which should lead to a phase lag of - $180 \ deg$ and not more. This continuous increase in phase lag is due to a time delay.

Measuring the time delay and subtracting it from the system response allowed a good match between the model and the measurement for higher frequencies.

5.2.2. Voltage mode

Figure 5.9 shows the open loop block diagram from Figure 3.3 rearranged. It should be mentioned that the amplifier gain is not equivalent anymore with the earlier derived amplifier gain in current mode. Now, the input to the amplifier is a voltage. The current, which finally flows through the actuator coil, depends on the back EMF, the eddy current damping and the coil impedance.



Figure 5.9: Simplified OL block model in voltage mode

This block collocation shows the difference between the current mode and the voltage mode. The input to the amplifier in current mode is amplified and sent as a current directly through the actuator coil. In the voltage mode the input voltage is amplified with a different gain. The current through the amplifier is then determined by both the mechanical and electrical system parameters, as shows the block represented as the actuator impedance. The latter part of the block diagram relates the current through the actuator coil to the applied displacement. This part looks equivalent to the earlier presented model in current mode. However, there is one significant difference. In the current mode the system damping was due to the mechanical damping of the actuator only. Any current losses in the coil due to eddy

current damping were compensated for by the amplifier, so that the desired current flow was guaranteed. Now, the damping includes both the mechanical damping of the actuator as well as the eddy current damping K_d in the coil. In other words the current through the coil may result in a low actuator displacement, providing that there might be energy losses in form of heat. The eddy current damping can therefore be approximated by relating again the actuator displacement to the current.

5.2.2.1. TF displacement / current monitor output

Figure 5.10 shows the differences to the earlier displacement/current FRF in current mode (Figure 5.4).



Figure 5.10: Displacement/current in voltage mode

The damping includes the eddy current damping and no amplifier gain can be measured anymore as the current monitor output is the current which directly flows through the actuator coil and does not pass the amplifier anymore.



Figure 5.11: FRF (LVDT/current) in voltage mode

Figure 5.11 shows that due to the eddy current damping in voltage mode the resonance peak decreases. The curve match is possible for an overall damping of

$$c_{total} = c_m + K_d = 11\frac{Ns}{m}$$
(5.8)

If the amplifier gain in current mode is taken into account, then the two measured frequency response functions, relating the actuator displacement to the current flow, can be compared. Figure 5.12 shows the lower frequency peak in voltage mode due to the additional electrical damping. While for low frequencies the mechanical damping might be dominant, the eddy current damping will determine the high frequency damping due to its frequency dependence. However it will be reasonable and sufficiently accurate to model the system damping according to equation (5.8).



Figure 5.12: Comparison of measured frequency response in current and voltage mode

5.2.2.2. TF current monitor output / voltage

The remaining unknown parameters are the inductance L of the actuator coil and the amplifier gain G_v in voltage mode. In the previous paragraph the latter part of Figure 5.9 has been measured. Now, by relating the current monitor output to the input voltage signal, the first section will be examined.



Figure 5.13: FRF (current/voltage)

The low frequency response allows a conclusion about the amplifier gain, which could be approximated as $G_v = 25$ from the measurement. For very high frequencies the transfer function can be approximated as:

$$H_{iv}(s \to \infty) = \frac{G_v}{Ls} \tag{5.9}$$

This shows that for growing frequency the coil inductance is the determining factor in reducing the system response. The coil inductance L could be approximated from the measured magnitude plot in the high frequency range as L = 0.08 H.

All parameters from the earlier shown model could be derived. In the following measurements their accuracy and reliability will be tested. The transfer function is rewritten below with *c* representing the total damping including both the mechanical and eddy current damping.

$$H_{vi,ol}(s) = G_v \frac{ms^2 + cs + k}{Lms^3 + (cL + mR)s^2 + (Bl^2 + kL + cR)s + kR}$$
(5.10)

Figure 5.14 shows that the model and the measurement are relatively close for the derived parameters. The continuous phase drop in the measurement has earlier been shown as a consequence of the time delay.



Figure 5.14: Current/Voltage FRF modeled and measured

5.2.2.3. TF displacement/voltage

The following test finally checks the accuracy of the derived open loop transfer function in equation (3.20), which relates the voltage input to the applied actuator displacement. All parameter units have been shown in the metric system. The open loop transfer function in equation (3.20) will therefore yield the displacement in *meters* for an input voltage in *volts*. As the LVDT measures the displacement in *inches* a conversion factor *conv* from meters to inches will be introduced into the equation in addition to the amplifier gain G_v .

$$H_{ol}(s) = \frac{x}{u} = \frac{G_v \operatorname{conv} Bl}{Lms^3 + (cL + mR)s^2 + (Bl^2 + kL + cR)s + kR}$$
(5.11)

Again with $s = j\omega$ the magnitude and phase yield

$$Mag_{ol}(\omega) = \frac{G_{v} conv Bl}{\sqrt{[kR - (cL + mR)\omega^{2}]^{2} + [(Bl^{2} + kL + cR)\omega - Lm\omega^{3}]^{2}}}$$

$$\theta_{ol}(\omega) = arcTan[\frac{mL\omega^{3} - (kL + Bl^{2} + cR)\omega}{kR - (mR + cL)\omega^{2}}]$$
(5.12)

The measured and modeled FRF are close (Figure 5.15). The actuator model is therefore valid and the derived parameters are sufficiently accurate.



Figure 5.15: Open loop FRF (displacement/voltage) measured and simulated

5.2.3. System uncertainties

The actuator parameters were derived measuring different time and frequency responses. The good match between the modeled and measured FRF indicates that the model captured all poles and zeros of the system in the low frequency range, i.e. the dynamics of the electromagnetic actuator are fully understood. However, in none of the different measurements a perfect match could be achieved between the model and the measurement. In this paragraph, potential factors are discussed which cause the deviation between the model and the measurement.

5.2.3.1. Time delay

The time delay has been shown as the major reason why the phase lag continuously increases. It does not affect the magnitude plot and does not introduce any more zeros or poles to the system. Chapter 3 has already introduced the transfer function affected by a time delay as $H_{\lambda} = H e^{-\lambda s}$. This indicates that the time delay grows for increasing frequency.

5.2.3.2. Eddy current damping

The effect of the eddy current damping has been shown by comparing the displacement/current FRF in voltage and current mode. It has been modeled as a constant which adds up with the mechanical damping of the actuator. In reality however, the eddy current damping is highly frequency dependent [44, 56]. It is probably one of the biggest factors explaining differences between the model and the measurement, especially for growing frequencies. Also the eddy current damping is not visible the same way for all the frequency response functions. It can cause some change in the current flow, which would be visible both in the current monitor output and the LVDT measurement. However, it also leads to pure energy loss in form of heat, which is only visible in the LVDT measurement but not in the current monitor output.

5.2.3.3. Amplifier poles and zeros

The amplifier has been treated as a proportional gain for both current and voltage mode. In reality the amplifier has its own dynamics which introduce poles and zeros to the system [13, 20, 42]. Those poles and zeros however are above the frequency range which has been measured. Hence, it is justified to assume the amplifier as a proportional gain only.

5.2.3.4. Coil impedance

The coil impedance has been modeled as a resistor in series with an inductive coil. In reality however, the coil impedance is not only linearly frequency dependent. This means that for growing frequencies the model becomes less accurate. It is very
difficult to capture the coil impedance accurately, however and the approximated inductance L can be seen as sufficiently accurate for all the different measurements.

5.2.3.5. Nonlinearity of rubber bands

It often turned out that for the same kind of test, the amplification factor in the frequency response varied depending on the input magnitude. As for both a large and small input good coherence could be achieved, it is possible that the actuator acts nonlinearly. One major reason is the nonlinearity of the rubber bands.

5.2.3.6. LVDT offset

Due to the friction of the rubber bands it is difficult to find a zero offset for the LVDT. After a movement the actuator will usually not go back to the initial point before the test. The rubber bands deform and slip over the bearings and sometimes center the actuator at slightly different positions after the test. This makes it hard to find a zero offset for the LVDT.

5.2.3.7. LVDT and actuator friction

The LVDT could not be centered without any friction. The actuator movement is therefore slightly inhibited by the friction within the LVDT device. Moreover there is friction in the actuator which is not captured by the transfer functions.

5.2.3.8. Soil-structure-interaction

The actuator and the structure are fixed to a steel plate and are therefore relatively fixed to each other. However the steel plate is not bolted to the ground but lies on a

nonrigid table. There is though some interaction between the steel plate and the table, in general terms a soil-structure interaction. An analysis of this effect however concluded, that no other setup must be chosen. Even though there is some distortion of the results due to the table especially in the eigenfrequency range of the actuator, it turned out as small enough.

5.2.3.9. Sampling frequency

The different frequency response measurements require input about the sampling frequency and number of data points. By this discretization process the continuous signal is sometimes not fully captured. Especially for high frequencies leakage and aliasing may influence the accuracy of the results.

5.2.3.10. Inconsistency of measurements

Most of the measurements vary slightly due to repetition, different location of the setup or even relocation of the cables. Temperature change due to increased use of the actuator and amplifier can affect the resistance and damping of the system. Magnetic interaction between the actuator and the measurement devices, as well as the cables has been discovered as a factor, which can influence the measurements, too. Likewise, noise affects the measurements especially for growing frequencies. With an attached load cell, the moving mass slightly increases. Resetting the rubber bands often leads to a slightly different actuator stiffness. All these factors are able to change the system response to a certain degree. It seems though justified to consider the derived actuator model as sufficiently accurate.

In this chapter all system parameters could be derived. The earlier derived transfer function allows for a good prediction of the real system behavior. It is thus justified to consider the system as fully understood and the derived transfer function and parameters as accurate enough to be used in further models.

6. FEEDFORWARD

Stability issues require the control of the actuator impedance as a function of the attached structure. This way the coupled actuator-structure system reveals the desired stable dynamics, which are represented by sufficiently damped poles in the frequency domain. Chapter four outlined that an additional feedforward input can improve the actuator tracking while maintaining the required impedance.

In this study, the feedforward is designed to compensate for the static deformation of the actuator due to the interaction force. In other words, while the dynamic response of the actuator is still affected by the interaction force, the feedforward fully compensates for the actuator-structure-interaction in a static state, as schematically shown in Figure 6.1.



Figure 6.1: Full dynamic compensation



Figure 6.2: Feedforward compensation for static deformation

In displacement control this requires as an additional voltage input equivalent to the static displacement error (Figure 6.2). It is practically impossible to compensate for the interaction dynamically, as this would require the prediction of the interaction force at the moment the feedforward signal must be applied. The feedforward which would compensate completely for the actuator deformation due to the interaction force is derived in equation (6.1).

$$ff H_{rx} = F H_{fx} = x_F$$

$$ff = \frac{H_{fx}}{H_{rx}} F = \frac{R + Ls}{Bl K_p} F = \frac{R}{Bl K_p} F + \frac{Ls}{Bl K_p} F$$
(6.1)

This transfer function is improper. The feedforward is therefore designed to compensate for the actuator-structure-interaction only statically, by applying only the static term $ff = RF / Bl K_p$ in equation (6.1).

The following feedforward schemes all compensate for the interaction statically but are derived in different ways. This way they differ in regard to the resulting actuator impedance, the robustness towards disturbances and their applicability for nonlinear structures. In the first scheme the feedforward derives from the reference displacement (FF_r), in the second one from the applied actuator displacement (FF_x), and in the third one directly from the interaction force (FF_{if}). In the FF_r the actuator impedance can only be varied by adjusting the gain. In the FF_x or FF_{if} , softening takes place even for a constant gain due to the external feedforward loop.

6.1. FF derived from reference input (FF_r)

For well-known structure stiffness, the feedforward can be derived from the reference input. Under assumed perfect actuator tracking, the interaction force equals the product of the reference displacement *r* and the linear structure stiffness *k*. This requires the additional feedforward input $ff_r = (kR/BlK_p)r$.



Figure 6.3: Simplified reference derived model

Figure 6.3 shows the corresponding amplification of the actuator input. This way, the feedforward does not change the system dynamics or the poles of the system, stiffening and softening of the actuator solely results from adjusting the proportional gain. The higher the structure stiffness, the softer the actuator and lower the proportional gain must be in order to avoid instability (Figure 6.4). The low impedance consequently results in a higher actuator deformation by the interaction force and requires a higher feedforward compensation.



Figure 6.4: Required gain adjustment for varying stiffness

The root locus plot represents the electromagnetic actuator with the derived parameters for free motion and a high stiffness of $k = 10^5 N/m$, which means a stiffness ratio of $K_{ratio} \approx 10$. This root locus is the same for the uncompensated actuator and for the FF_r. It is similar to the shown root locus plot in chapter 3, where the problem of the higher stiffness has been shown for nondimensional parameters.

The required actuator softening has been explained with the instability problems for stiff structures. The required actuator stiffening for soft structure has only been justified briefly with the actuator phase lag. How the poles' location affect the phase lag will now be solved analytically.

If the feedforward is derived directly from the reference input, then the phase angle equals the phase angle of the original system without feedforward. The closed loop transfer function without the feedforward gain was:

$$H_{cl}(s) = \frac{x}{r} = \frac{K_p Bl}{mLs^3 + (mR + cL)s^2 + (kL + cR + (Bl)^2)s + kR + K_p Bl}$$
(6.2)

With $s = j\omega$ (6.2) can be split into a real and an imaginary part.

$$H_{cl}(j\omega) = \frac{K_{p}Bl}{[K_{p}Bl + kR - (mR + cL)\omega^{2}] + j[(kL + (Bl)^{2} + cR)\omega - mL\omega^{3}]} = K_{p}Bl \frac{[K_{p}Bl + kR - (mR + cL)\omega^{2}] - j[(kL + (Bl)^{2} + cR)\omega - mL\omega^{3}]}{[K_{p}Bl + kR - (mR + cL)\omega^{2}]^{2} + [(kL + (Bl)^{2} + cR)\omega - mL\omega^{3}]^{2}}$$
(6.3)

The phase angle then results as

$$\theta_{cl} = -arcTan[\frac{(kL + (Bl)^2 + cR)\omega - mL\omega^3}{K_{\nu}Bl + kR - (mR + cL)\omega^2}]$$
(6.4)

Equation (6.4) shows that the phase lag increases for lower stiffness and constant gain. The increase in the phase lag can only be compensated for by a higher gain. Likewise this means that a higher structure stiffness decreases the problematic phase lag. The requirement of a softer actuator in response to a stiff structure is thus also in accordance with the phase lag and not only justified by the stability bounds. For infinite structure stiffness and a finite gain, the phase lag yields towards the limit in equation (6.5).

$$\lim_{k \to \infty} \theta = \arctan\left[\frac{mL\omega^3 - (kL + (Bl)^2 + cR)\omega}{K_pBl + kR - (mR + cL)\omega^2}\right] = \arctan\left[-\frac{L\omega}{R}\right]$$
(6.5)

This means that there is always a phase lag present, unless the gain is increased infinitely, which would mean actuator instability. The actuator phase lag is therefore another challenge in dynamic testing and in particular in hybrid testing. This will be discussed in detail in chapter 7.

6.1.1. Advantages and disadvantages

The shown scheme requires an exact knowledge of the system parameters. If the structure stiffness is not known well, then an over- or undercompensation of the actuator deformation due to the interaction force occurs. The feedforward however does not relocate the poles of the system, so that a badly computed feedforward gain cannot destabilize the system, but only leads to inaccuracies. A nonlinear stiffness would require a constant adjustment of the proportional gain, which is not applicable as long as the nonlinear structure stiffness is unknown. The biggest disadvantage is however, that the scheme cannot actively increase system damping. The presented instability problems due to stiff structures remain, as the model can only avoid that the poles shift even closer towards instability than they already are. Equation (6.6) repeats the earlier derived system impedance. It confirms that a lower proportional gain can only partially compensate for an increase in the structure stiffness.

$$\hat{K} = \left| \frac{F_{ext}}{x_{ext}} \right| = \frac{Lms^3 + (mR + cL)s^2 + (cR + Bl^2)s}{R + Ls} + \frac{BlK_p}{R + Ls} + k$$
(6.6)

6.2. FF derived from applied displacement (FF_x)

If the feedforward derives from the applied displacement according to Figure 6.5, then the system dynamics are actively changed. Figure 6.5 models the interaction force as the product of the applied displacement x and the structure stiffness k. For a linear structure stiffness, the computed and measured force will be equivalent. The feedforward is this measured or computed force, divided by the actuator gain in direct current.



Figure 6.5: Feedforward derived from applied displacement (FF_x)

$$H_{ol,ffx} = \frac{1}{s} \frac{Bl}{Lms^{2} + (cL + mR)s + (cR + kL + (Bl)^{2})}$$

$$H_{cl,ffx} = \frac{Bl K_{p}}{Lms^{3} + (cL + mR)s^{2} + (cR + kL + (Bl)^{2})s + Bl K_{p}}$$

$$K_{pmax,ffx} \leq \frac{(cR + (Bl)^{2})(cL + mR) + k(cL + mR)}{Bl Lm} = K_{pmax} + \frac{kR}{Bl L}$$
(6.7)

The open loop and closed loop transfer functions, as well as the stability limits from the proportional gain change correspondingly. Equation (6.7) shows a pole at the origin and a higher allowable proportional gain in comparison to the uncompensated model. Those properties are also illustrated in the root locus plot in Figure 6.6.



Figure 6.6: RL with FF_x

Both for free motion and a the high structure stiffness of $k = 10^5 N/m$, the complex conjugate pairs of poles start from the same distance to the imaginary axis. In comparison to the earlier presented scheme this means, that even for extremely high structure stiffness, the system poles can keep a safe distance to the imaginary axis. In addition to this big advantage however, Figure 6.6 also shows a real pole at the origin for every structure stiffness. For higher stiffness the system additionally becomes more robust towards the proportional gain. This is visible in Figure 6.6, as the stiff root locus path crosses the imaginary axis for a much higher gain. I.e. the root locus is slower for higher structure stiffness and a higher proportional gain. The outer feedback loop softens the system automatically. Seraji and Colbaugh [45] showed that an outer feedforward loop is able to soften a system effectively even though the inner feedback system is very stiff.

In comparison to the uncompensated model of the FF_r, the FF_x shows increased stability but also a higher phase lag for the same gain. For $s = j\omega$, the closed loop transfer function in equation (6.7) is rewritten as

$$H_{cl,fj\hat{x}}(j\omega) = \frac{K_{p}Bl}{[K_{p}Bl - (mR + cL)\omega^{2}] + j[(kL + cR + (Bl)^{2})\omega - mL\omega^{3}]} = K_{p}Bl \frac{[K_{p}Bl - (mR + cL)\omega^{2}] - j[(kL + cR + (Bl)^{2})\omega - mL\omega^{3}]}{[K_{p}Bl - (mR + cL)\omega^{2}]^{2} + [(kL + cR + (Bl)^{2})\omega - mL\omega^{3}]^{2}}$$
(6.8)

This shows again that the phase angle in equation (6.9) depends on all parameters: the actuator characteristics, the proportional gain, the structure stiffness and the frequency of the external source.

$$\theta_{ffx} = -arcTan[\frac{(kL+cR+(Bl)^2)\omega - mL\omega^3}{K_pBl - (mR+cL)\omega^2}]$$
(6.9)

In contrast to equation (6.5) the phase lag now increases for growing structure stiffness and constant proportional gain. This requires that the proportional gain must be increased for higher structure stiffness.

As this feedforward actively determines the pole location, system uncertainties now do not only affect accuracy but also stability. In the following the assumed structure stiffness k_e differs from the real stiffness k by an error factor E_F , $k_e = E_F k$.

I.e. the system is overcompensated for $E_F > 1$ and undercompenated for $E_F < 1$. The open and closed loop transfer functions then differ from the original.

$$H_{ol,ffx,E_{F}} = \frac{Bl}{Lms^{3} + (cL + mR)s^{2} + (cR + kL + (Bl)^{2})s + (1 - E_{F})kR}$$

$$H_{cl,ffx,E_{F}} = \frac{BlK_{p}}{Lms^{3} + (cL + mR)s^{2} + (cR + kL + (Bl)^{2})s + (1 - E_{F})kR + BlK_{p}}$$
(6.10)

If $E_F < 1$, then due to the undercompensation less softening takes place and the upper limit for the proportional gain is lower than for accurate compensation. However the allowable limit will never drop below the limit of the FF_r or the uncompensated scheme. If the system is overcompensated ($E_F > 1$), then there is a lower bound for the proportional gain.

$$K_{p,ffx,E_F} \ge \frac{kR}{Bl}(E_F - 1) \tag{6.11}$$

This is reasonable as the real pole moves into the right half plane. A minimum gain is needed to stabilize the system, as shown in Figure 6.7. Both stability bounds however are very unlikely to happen as long as the structure stiffness is not estimated completely wrong.



Figure 6.7: RL for K_p with FF_x and E_F =5 (overcompensation)

6.2.1. Advantages and disadvantages

This scheme reveals two great advantages in comparison to the FF_r . Any structure stiffness can be tested safely, and in addition, impedance control does not necessarily require a gain adjustment, but occurs automatically due to the feedforward loop. The system impedance for the FF_x is shown below.

$$\hat{K}_{FF_{x}} = \left| \frac{F_{ext}}{x_{ext}} \right| = \frac{Lms^{3} + (mR + cL)s^{2} + (cR + Bl^{2})s + BlK_{p}}{R + Ls} - \frac{R}{R + Ls}k + k$$
(6.12)

In contrast to the uncompensated system of the FF_r , equation (6.12) shows an additional term which lowers the actuator impedance automatically for a stiffer structure.

Like in the previous feedforward scheme, a miscalculated structure stiffness leads to a systematic over- or undercompensation of the actuator deformation due to the structure. For undercompensation less softening takes place, and the upper limit for the proportional gain is lower than for accurate compensation. However, the allowable limit will never drop below the limit of the FF_r or the uncompensated scheme. If the system is overcompensated, then there is a required lower bound for the proportional gain, which becomes more critical for increasing overcompensation.

A drawback of this scheme can be that a softer actuator has a higher phase lag. If the structure stiffness exceeds the estimated stiffness extremely, then the applied gain may be too low to shift the real pole enough into the left half plane, in other words too much softening takes place. Consequently, a higher actuator delay would lead to negative damping in hybrid testing. This will be discussed in more detail in chapter 7.

Both presented feedforward schemes can eliminate the static actuator deformation due to the interaction force, but are only valid for linear and known structure stiffness. It can therefore be reasonable to eliminate the structure stiffness from the feedforward by directly measuring the interaction force at the actuator-structure interface.

6.3. FRF over the whole actuator bandwidth with FF

All shown feedforward schemes are designed to compensate for the interaction force in DC. This design improves the actuator tracking not only in DC as showed the improved tracking of the square wave, but over the whole lower frequency range. Figure 6.8 shows the simulated frequency response of the three models for a high stiffness of $k = 10^4 N/m$ and a gain, which is half the maximum allowable gain in free motion.



Figure 6.8: Simulated FRF with K_p=0.5K_{pmax} for high stiffness and all schemes

The feedforward improves the reference tracking for all frequencies below the cutoff frequency. As explained above, the FF_x is more robust towards the proportional gain. For the same applied gain this means, that the time response due to the real pole in the FF_x scheme is not as quickly damped out as in the FF_r or in the uncompensated scheme. For increasing frequencies, it therefore shows a higher phase lag and worse tracking capabilities.

6.4. FF derived from interaction force (FF_{if})



Figure 6.9: Block diagram with force derived feedforward

Figure 6.9 shows the block diagram, where the feedforward is derived directly from the measured interaction force. If the structure stiffness is linear, then the dynamics of this scheme are equivalent to the FF_x . If the load cell is well calibrated and the actuator properties well-known, then an over- or undercompensation due to feedforward is impossible. Moreover this scheme can be applied, if the structure stiffness is nonlinear or even unknown.

Another advantage is that this scheme partially compensates for force disturbances at the actuator-structure interface which are not due to the structure stiffness. The actuator displacement of the compensated model in Figure 6.9 is

$$x = \frac{Bl K_{p}r - LsF}{Lms^{3} + (cL + mR)s^{2} + (cR + (Bl)^{2})s + Bl K_{p}}$$
(6.13)

Even though the static actuator-structure-interaction is compensated for, dynamically the interaction force still causes the following actuator deformation.



Figure 6.10: Deformation due to force disturbance



Figure 6.11: Relative error compensation by FF

Figure 6.10 simulates the actuator deformation due to a unit force for the original, uncompensated system, and the system with force derived feedforward for the earlier presented actuator parameters, and a proportional gain of half the maximum allowable gain. The feedforward significantly improves the results, particularly for lower frequencies. The relative compensation by feedforward is independent of the gain and plotted in Figure 6.11. For both models, the system is less sensitive to disturbances beyond the cut-off frequency of the actuator. This is due to the actuator dynamics, which act like a low pass filter.

6.5. Comparison of the different schemes

In the following, the differences of the shown feedforward schemes are summarized again. If the feedforward is derived from the reference input, then the system is not changed in regard to stability and phase lag. Deriving the feedforward from the applied displacement or the interaction force adjusts the actuator impedance to a varying structure stiffness. For stiff structures the advantage of the actuator softening and correspondingly the higher system damping can be seen in both more stability but also a higher phase lag.

FF _x and FF _{if}	Uncompensated system and FF _r
Initial location of complex conjugate pair of poles shifted into the LHP.	Initial location of complex conjugate pair of poles unchanged.
Initial distance of poles to imaginary axis (~settling time) is independent of structure stiffness.	Distance decreases with growing structure stiffness.
High stability margin also for high stiffness	Low stability margin for high stiffness.
Complex conjugate pair is only dominant for high gains.	For high gains OR/AND high stiffness the complex conjugate pair is always dominant.
Gain reduction dramatically increases the phase lag.	Gain reduction only problematic for low structure stiffness.
The outer feedforward loop softens the system automatically.	The outer feedforward loop cannot soften the system, a gain reduction is therefore necessary.

6.6. Test results

The different feedforward schemes will now be applied in laboratory tests. In order to model a structure stiffness, two L-angles are fixed to the steel plate (Figure 6.12). A brass bar is then clamped horizontally, as shown schematically in Figure 6.13. This setup is chosen, because no bending will be applied to the load cell nor friction will introduce noise into the system when the beam is deformed.



Figure 6.12: Setup with modeled stiffness



Figure 6.13: Equivalent structural model



Figure 6.14: Actuator-structure test setup

Figure 6.14 and Figure 6.15 show that the load cell is fixed to the end of the actuator and is pinned to the center of the brass bar. The modeled stiffness of the brass bar is computed based on the assumed and measured properties of the bar and is likewise measured with the LVDT and load cell. It is highly nonlinear as it stiffens up for higher applied displacement and due to the way it is clamped also shows some difference between tension and compression. All tests in this and the following chapter are performed with a 0.032 inches thick brass bar. Its stiffness has been measured and is represented with an approximation function in Figure 6.16.



Figure 6.15: Actuator-structure attachment



Figure 6.16: Nonlinear stiffness of brass bar

This stiffness sums up with the actuator rubber bands to the overall stiffness which finally resists the actuator movement. In the following example, the FF_r and FF_x will hence be compared. The maximum gains for the derived parameters are:

$$K_{p,FF_{r}}\left[\frac{in}{V}\right] < \frac{BlR}{L\,conv\,G_{v}} + c\frac{Bl^{2}L + kL^{2} + mR^{2} + LRc}{Bl\,Lm\,conv\,G_{v}} \approx 1.4 + 1.5 * 10^{-5} \,k$$

$$K_{p,FF_{x}}\left[\frac{in}{V}\right] \le \frac{(cL + mR)(cR + Bl^{2} + kL)}{Bl\,Lm\,conv\,G_{v}} \approx 1.4 + 1.53 * 10^{-4} \,k$$
(6.15)

6.6.1. Experiment 1

In the first experiment, the frequency response over the whole actuator bandwidth is measured for the uncompensated model and the feedforward compensated schemes, applying a relatively high stiffness ($\rho > 0.5$) and a gain of $K_p = 0.8K_{pmax}$. A multisine input voltage with low amplitude is applied to decrease the effect of the structure nonlinearity.



Figure 6.17: Improved tracking by FF over the whole actuator bandwidth

Figure 6.17 shows that both feedforward schemes improve the actuator tracking over the whole actuator bandwidth. However, it should be noted that the FF_{if} leads to a higher phase lag due to the softening of the actuator.

6.6.2. Experiment 2

In the following example the actuator is inhibited by the structure stiffness from Figure 6.16 in compression, while in tension only the inherent rubber bands affect its movement. This experimental setup is schematically shown in Figure 6.18. This time, the load cell is not fixed to the brass bar. For negative displacement the actuator is not in contact with the brass bar and only experiences the stiffness of the rubber bands. For positive displacement the brass bar resists the movement of the actuator in addition to the rubber bands.



Figure 6.18: Actuator acting on nonlinear structure

For the commanded square wave the actuator approximately faces the stiffness k=5670N/m in compression and k=210N/m in tension. Figure 6.19 shows that for the chosen gain the actuator cannot track the signal in neither one of the two directions. While in tension the low stiffness only results in a low static error, in compression the large interaction force results in a large static error.



Figure 6.19: Bad tracking for no FF

This DC error is not present anymore, if one of the two feedforward schemes is applied. Figure 6.21 shows both the advantages and disadvantages of the FF_r . The feedforward eliminates the DC error, but after a few cycles the system goes unstable. Even though stable tracking occurred for the first cycles, some system uncertainty or disturbance destabilized the system. This outlines the big advantage of the displacement derived scheme (FF_x). The feedforward loop increases the system damping, leading to a stable response with lower overshoot, as visible in Figure 6.20.



Figure 6.20: Accurate and stable tracking with FF_x



Figure 6.21: Instability of FF_r

This example illustrates that only the FF_x scheme, or equivalently the FF_{if} , provide for enough damping for high structure stiffness.

The previous example showed that the FF_x successfully compensates for the actuator-structure-interaction in DC. Additionally, the external feedforward loop showed a stabilizing effect by controlling the actuator impedance in respond to the structure. Earlier it has been mentioned that for known structure stiffness and no further force disturbance, the FF_x and FF_{if} are equivalent schemes. The differences become obvious when the force disturbance at the actuator-structure interface cannot be computed anymore accurately. This happens for instance for a nonlinear or unknown structure stiffness or for other force disturbances, which are not a function of the applied actuator displacement.

6.6.3. Experiment 3

In the following experiment the tested structure is a magnetorheological (MR) friction damper [30] (Figure 6.22 a) and b)), which is only slightly velocity

dependent and resists the actuator movement with an approximately constant force of 9*N*.



Figure 6.22: MR friction damper attached to actuator



Figure 6.23: measured FRF with friction damper

The measured FRF for the uncompensated and feedforward compensated systems in comparison to the actuator movement in free motion (no damper attached) are shown in Figure 6.23. The FRF with feedforward is almost the same as the FRF of the actuator in free motion, indicating that feedforward is effective in compensating for

actuator-structure-interaction. The resonance peak of the compensated frequency response occurs for a slightly lower frequency than that of the actuator in free motion, because the MR friction damper has some moving mass. The improved actuator tracking is now shown in the time domain for a sine wave input and a frequency of f = 0.1 Hz. Figure 6.24 shows that the damper resistance leads to a bad tracking, which for this low frequency can be improved significantly by feedforward.



Figure 6.24: Improved tracking with FF and magnetorheological friction damper

6.7. Comparison between FF and PID controller



Figure 6.25: PID control

Figure 6.26: PID controller

For PID feedback control, the input signal to the actuator is the displacement error, hence the difference between the reference displacement r and the applied displacement x, in series with a PID controller (Figure 6.25). A PID controller consists of a proportional, an integral and a derivative part (Figure 6.26). The proportional gain K_p controls the natural frequency. It decreases the rise time and reduces the error response due to disturbances. However it has been shown that the proportional gain cannot eliminate the DC error and leads to instability of the system if the critical limit is exceeded.

The integral gain K_I reduces the DC error or can even eliminate it. Like the proportional gain, however, it increases the overshoot of the system response and in the limit leads to instability. The derivative gain K_d can increase the damping. This way the overshoot decreases and better stability is achieved. The effect of the derivative control term depends on the rate of change of the error. This is difficult to design so that the derivate controller is often not used. Another significant disadvantage is, that noise in the system can be amplified by a high derivative gain.



Figure 6.27: Simulated FRF for feedforward and integral control

Figure 6.27 shows the simulated frequency response, using an integral controller with the magnitude K_i of half the maximum allowable integral gain. Like the feedforward models, the integral controller can likewise compensate for the DC error for lower frequencies. However there are important differences between the feedforward and integral controller in regard to stability, which become increasingly important for higher structure stiffness. While feedforward schemes (FF_x of FF_{if}) have been shown able to increase the system damping by shifting the complex conjugate pair of poles into the left half plane, the integral controller adds another real pole into the system, which cannot increase the system stability. Figure 6.28 shows the root locus for increasing integral gain for the same system parameters as applied before, i.e. the same proportional gain and structure stiffness.

The integral controller introduces another real pole and cannot move the complex conjugate pair of poles away from the imaginary axis. In contrary, increasing the integral gain shifts the complex conjugate pair of poles closer to the imaginary axis and decreases the system damping even more.



Figure 6.28: Root locus for increasing integral gain



Figure 6.29: Root locus for growing derivative gai

The derivative controller adds another zero into the system and can actively increase the system damping. Figure 6.29 shows that for the same parameters the derivative gain can move the complex conjugate pair of poles further away from the imaginary axis. Simultaneously however, the real pole shifts towards the right and becomes dominant very quickly leading to a slow response and large phase lag. Figure 6.29 shows the location where for this specific case the poles cross. In addition, the derivative controller reveals further disadvantages and is therefore not used frequently. For example, some system noise can be amplified and introduce even higher inaccuracies to the system.

Summarizing the effect of a PID controller, it introduces another pole and two other zeros to the system (equation (6.16)). A very good design of all parameters can improve both the stability and tracking capability of an actuator. The disadvantage is however, that a good design of the gains is only valid for a specific structure. As the control system does not adjust to changing structure stiffness, the basic law of impedance control is broken, and simultaneous accuracy and stability are not possible anymore.

$$H_{cl,PID} = \frac{x}{r} = Bl \frac{K_d s^2 + K_p s + K_i}{Lms^4 + (cL + mR)s^3 + (Bl^2 + Bl K_d + kL + cR)s^2 + (Bl K_p + kR)s + Bl K_i}$$
(6.16)

6.8. Actuator resistance of higher order dynamics

It has been shown that the force derived feedforward can successfully compensate for interaction forces due to springs or force disturbances such as friction. In the following it will be analyzed if this scheme is also applicable if a dynamic system is attached to the actuator. If the actuator is attached to a shake table or damper, then the system poles will be changed in another way than by a spring. The mass, damper and stiffness of the actuator (m, c, k) are distinguished from the mass, damping and stiffness (M, C, K) of the test structure in Figure 6.30.



Figure 6.30: Actuator resisted by spring-mass-dashpot

6.8.1. Uncompensated system dynamics

If no feedforward is applied, then the attached mass, damper and stiffness have the same effect on the system dynamics than the internal properties of the actuator, i.e. the moving mass, mechanical and eddy current damping and the stiffness of the rubber bands. This way the corresponding properties add up in the open and closed loop transfer function.

$$\begin{split} H_{ol,MCK} &= \\ \frac{Bl}{(m+M)Ls^3 + ((m+M)R + (c+C)L)s^2 + ((k+K)L + (c+C)R + (Bl)^2)s + (k+K)R} \\ H_{cl,MCK} &= \\ \hline \\ \frac{Bl K_p}{(m+M)Ls^3 + ((m+M)R + (c+C)L)s^2 + ((k+K)L + (c+C)R + (Bl)^2)s + (k+K)R + Bl K_p} \\ K_{p,MCK} &\leq \frac{BlR}{L} + (c+C)\frac{Bl^2L + (k+K)L^2 + (m+M)R^2 + LR(c+C)}{Bl L(m+M)} \\ \end{cases} \end{split}$$

The maximum proportional gain increases due to the applied structure stiffness K and particularly due to the damper C. The mass M however is shown as a lowering factor.



Figure 6.31: RL for different attached masses

The root locus in Figure 6.31 basically shows a decrease in the eigenfrequency of the system. The system response gets slower and the settling time increases. For infinite high mass, the gain converges towards the limit in equation (6.18).

$$K_p(M \to \infty) < \frac{BlR}{L} + \frac{cR^2}{BlL}$$
(6.18)

Increased damping shifts the complex conjugate pole into the LHP and increases the dominance of the real pole (Figure 6.32).



Figure 6.32: RL for different attached dampers

6.8.2. Compensated system dynamics

It is now of interest how the shown force derived feedforward scheme performs with an attached dynamic system. The interaction force is now modeled as a function of not only the actuator displacement but also the velocity and mass.

$$ff_{MCK} = \frac{R}{K_p Bl} (Ms^2 + Cs + K)$$

$$H_{cl,MCK,FF} = \frac{Bl K_p}{(m+M)Ls^3 + (mR + (c+C)L)s^2 + ((k+K)L + cR + (Bl)^2)s + kR + Bl K_p}$$
(6.19)
$$K_{p,MCK,FF} \le \frac{L}{Bl L(m+M)} (K(cL + mR) + C(kL + cR + (Bl)^2) + KC - MkR)$$

Like in the uncompensated model the stiffness and damper allow for a higher gain, while the mass decreases it. In fact, if no additional damping or stiffness is applied, then there is a maximum allowable mass.

$$M < (Bl^{2}cL + Bl^{2}mR + ckL^{2} + cmR^{2} + LRc^{2})/(kLR)$$
(6.20)



Figure 6.33: RL with FF for varying attached masses

The increasing mass lowers the stability bounds and even leads to instability quickly. This concludes that the force derived feedforward is not applicable for interaction forces due to inertia. This is different for an applied damper. The allowable gain is increased and more stability achieved.

$$K_{p} < \frac{Bl^{2}cL + Bl^{2}mR + ckL^{2} + cmR^{2} + LRc^{2}}{Bl L m} + C\frac{kL + cR + Bl^{2}}{Bl (m + M)}$$
(6.21)

Figure 6.34: RL with FF for varying attached dampers

Real Axis

Figure 6.34 shows that the feedforward shifts the poles into the LHP and increases the system damping even further. This is not necessarily desired as already the uncompensated model has increased system damping. Impedance control requires that the soft structure, here the damper, requires a stiff actuator. However the shown root locus becomes more sensitive for the gain as well. This means that for the applied gain the poles are much closer to the imaginary axis than without feedforward.

It thus can be summarized that feedforward should not be used to compensate for interaction forces due to any inertia force. It is however useful to compensate for a damper force. The system is more stable than in free motion for the same gain and more accurate than the uncompensated system with the attached damper. However, better control design requires the adjustment of the proportional gain to the attached damper. This rule is valid for the mass, damper and structure in general.

6.9. Summary

So far it has been explained that the actuator must deform in response to the interaction force. This way the actuator has a certain impedance, which is required to adjust to a changing structure stiffness. Feedforward has been shown as a tool to compensate for the actuator deformation effectively, by likewise maintaining its required impedance. A stiffer structure requires a softer actuator, which means that it deforms more due to the interaction force and a higher feedforward input is needed. This softening is equivalent to an increase in the actuator damping and stability. The disadvantage of the actuator softening has been shown in the higher phase lag especially for higher frequencies. In the root locus plot this could be seen by a more dominant real pole. The following chapter will discuss the applicability of feedforward in hybrid testing.

7. HYBRID MODEL

The actuator must deform in response to the interaction force. This way the actuator has a certain impedance, which is required to adjust to a changing structure stiffness. Feedforward has been shown as a tool to compensate for the actuator deformation, by likewise maintaining the requirement of the actuator impedance. In comparison to the displacement derived scheme, the force derived feedforward FF_{if} is applicable also for nonlinear or unknown structure stiffness. The reason is, that the feedforward loop softens the actuator in response to a higher structure stiffness and vice versa, even if the gain is kept constant. Simultaneously however, this can lead to a higher actuator phase lag, if the structure stiffness is highly underestimated and no gain adjustment applied. This was visible in the root locus plot where the higher stiffness led to more robustness towards the gain and consequently resulted in a more dominant real pole for the same gain.

In hybrid testing, the actuator delay, which results from the phase lag and other time delays in the system, has the effect of negative damping in the hybrid loop. For small delays, this negative damping can be approximated as the product of the structure stiffness k and the actuator delay δt ($c_{eq} = -k\delta t$) [53]. An underestimation of the structure stiffness therefore is particularly critical as the higher actuator phase lag in combination with the high structure stiffness then results in a high negative damping. It is therefore required to compensate for the possible actuator delay in particular for high structure stiffness. Prediction schemes [18, 37], numerical damping [16, 51,

53], as well as the [18]I-modification scheme [10] can reduce the harmful effect of the actuator delay. In the following analysis and laboratory tests, the I-modification scheme is shown capable to compensate for the actuator delay effectively, even though the structure stiffness is not known well.

7.1. Hybrid model



Figure 7.1: Chosen hybrid model

The simulations and tests will be performed for the following chosen system in Figure 7.1. In the hybrid simulation, the spring k_1 is modeled physically, while the rest of the structure is modeled numerically. The differential equation (7.1) shows, that the effect of the spring k_1 can be grouped with the external input and separated from the rest of the structure. In other words, the physical substructure is eliminated in the numerical substructure. Instead, the interaction force is taken as an external input. The accuracy of this force feedback depends on the tracking capabilities of the actuator.
$$\begin{bmatrix} m_{1} & 0 & 0 & c_{1}+c_{2} & -c_{2} & 0 & \mathbf{k}_{1}+k_{2} & -k_{2} & 0\\ 0 & m_{2} & 0 & -c_{2} & c_{2}+c_{3} & -c_{3} & -k_{2} & k_{2}+k_{3} & -k_{3}\\ 0 & 0 & m_{3} & 0 & -c_{3} & c_{3} & 0 & -k_{3} & k_{3} \end{bmatrix} \{\vec{x}\} = -\begin{cases} m_{1}\\ m_{2}\\ m_{2}\\ m_{2} \end{cases} \ddot{u}_{g}$$

$$\begin{bmatrix} m_{1} & 0 & 0 & c_{1}+c_{2} & -c_{2} & 0 & k_{2} & -k_{2} & 0\\ 0 & m_{2} & 0 & -c_{2} & c_{2}+c_{3} & -c_{3} & -k_{2} & k_{2}+k_{3} & -k_{3}\\ 0 & 0 & m_{3} & 0 & -c_{3} & c_{3} & 0 & -k_{3} & k_{3} \end{bmatrix} \{\vec{x}\} = -\begin{cases} m_{1}\\ m_{2}\\ m_{2}\\ m_{2} \end{cases} \ddot{u}_{g} - \begin{cases} \mathbf{k}, u_{1}\\ 0\\ 0 \end{cases} \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$(7 \ 1)$$

(7.1) where $\{\vec{x}\} = \{\vec{u}_1 \ \vec{u}_2 \ \vec{u}_3 \ \vec{u}_1 \ \vec{u}_2 \ \vec{u}_3 \ u_1 \ u_2 \ u_3\}^T$.

7.2. Actuator delay

In the previous chapter, the actuator phase lag has been derived for each feedforward scheme. It has been mentioned above, that the actuator delay is an inherent problem in hybrid testing as it introduces energy into the hybrid loop and can cause instability.

Assuming a sine input, the energy added into the system in one period T for a continuous system is the integral of the product of the feedback force with the correct (not delayed) displacement [18].

$$\delta E = \oint q dx = \int_{0}^{T} q \frac{dx}{dt} dt = \int_{0}^{T} (kx_{del}) \frac{dx}{dt} dt = \int_{0}^{T} kA \sin(\omega_0 t - \omega_0 \delta t) A\omega \cos(\omega_0 t) dt = \frac{1}{2} kA^2 2\pi \omega_0 \delta t$$
(7.2)

where q, k, A, ω_0 are the force over a cycle, the structure stiffness, the input and the natural frequency of the sine input correspondingly. For small delays, this energy increase can be approximated by the equivalent negative damping $c_{eq} = -k\delta t$ [53]. The following tests and simulations apply Newmark's constant average acceleration method, as it is unconditionally stable but has no numerical dissipation. This means that if no damping is applied for the structure $(c_1 = c_2 = c_3 = 0)$, then the least actuator delay will lead to instability.

While the upper allowable limit for the proportional gain does not depend on the hybrid model, but only on the actuator properties, the lower limit depends on the damping in the numerical substructure. In the previous chapter, higher actuator stability was reached by softening the actuator. In the hybrid loop this can now lead to the trade-off, that this softer and consequently slower actuator increases the negative damping. This means that depending on the numerical substructure and the stiffness of the physical substructure there is also a lower limit for the proportional gain.

The lower allowable limit cannot be computed accurately as the actuator in reality is a continuous system and the numerical scheme a discrete system. In the real laboratory test it is necessary to convert the digital signal into a continuous signal, before it is sent to the actuator. Likewise, the continuously measured actuator displacement and the interaction force are converted into a digital signal, before they reach the numerical model. An approximation can be computed if both systems are simulated as either discrete or continuous.

In the following, the three spring-mass-dashpots in Figure 7.1 are modeled as equivalent. If the numerical substructure is modeled as a continuous system and the physical substructure is assumed as linear, then the equilibrium of the hybrid system results in equation (7.3), where r is the actuator command and H_{cl} the closed loop actuator transfer function. U is the external force input to the numerical substructure

which results from both the ground acceleration and the load cell measurement, which is modeled as the applied actuator displacement multiplied with the structure stiffness. This implicit equation will be solved in the later examples for the corresponding parameters in order to estimate the lower allowable gain.

$$r = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} ms^{2} + 2cs + k & -cs - k & 0 \\ -cs - k & ms^{2} + 2cs + k & -cs - k \\ 0 & -cs - k & ms^{2} + cs + k \end{bmatrix} U$$

$$U = - \begin{cases} 1 \\ 1 \\ 1 \end{cases} m \ddot{u}_{g} - \begin{cases} 1 \\ 0 \\ 0 \end{cases} rk H_{cl}$$
(7.3)

7.3. Simulink model

The results of the hybrid model and the allowable stability limits for the actuator are first simulated with the previously discussed simulink models. In comparison to the real-time system, the simulink model is not affected by noise and approximates the actuator with the discretized transfer function.

7.3.1. Discretization

The algorithms solve for the displacements in discrete time steps. The actuator transfer function however has been shown as a continuous system. In the hybrid simulation the actuator transfer function is hence approximated as a discrete system. A bilinear approximation based on the trapezoidal rule of discrete equivalents, also known as Tustin's method [14], converts the continuous transfer function into the discrete transfer function for:

$$s \to \frac{2}{h} \frac{z-1}{z+1} \tag{7.4}$$

The earlier shown continuous transfer function then yields the discrete transfer function

$$H_{cl}(z) = \frac{K_p Bl h^3 (z^3 + 3z^2 + 3z + 1)}{b_1 z^3 + b_2 z^2 + b_3 z + b_4} = \frac{a_1 z^3 + a_2 z^2 + a_3 z + a_4}{b_1 z^3 + b_2 z^2 + b_3 z + b_4}$$
(7.5)

with

$$a_{1} = a_{4} = Bl K_{p}h^{3} \quad a_{2} = a_{3} = 3a_{1}$$

$$b_{1} = 2(Bl h)^{2} + Bl K_{p}h^{3} + 2Lkh^{2} + 8Lm + 4mhR + Rkh^{3}$$

$$b_{2} = 2(Bl h)^{2} + 3Bl K_{p}h^{3} + 2Lkh^{2} - 24Lm - 4mhR + 3Rkh^{3}$$

$$b_{3} = -2(Bl h)^{2} + 3Bl K_{p}h^{3} - 2Lkh^{2} + 24Lm - 4mhR + 3Rkh^{3}$$

$$b_{4} = -2(Bl h)^{2} + Bl K_{p}h^{3} - 2Lkh^{2} - 8Lm + 4mhR + Rkh^{3}$$
(7.6)

The DC for the continuous system is found for s = 0, for the discrete system this corresponds to z = 1. The gain of the two feedforward schemes has been shown as identical before. Equations (7.7) and (7.8) solve for the desired feedforward gains for the reference derived (ff_r) and displacement derived (ff_x) schemes again in order to compensate for the static actuator-structure interaction. As shown, they equal the gains of the continuous system.

$$ff_r H_{cl} = ff_r \frac{a_1 + a_2 + a_3 + a_4}{b_1 + b_2 + b_3 + b_4} = 1 \qquad \qquad ff_r = \frac{b_1 + b_2 + b_3 + b_4}{a_1 + a_2 + a_3 + a_4} = 1 + \frac{kR}{Bl K_p}$$
(7.7)

$$\frac{H_{cl}}{1 - ff_x H_{cl}} = 1 \qquad \qquad ff_x = \frac{1}{H_{cl}} - 1 = \frac{b_1 + b_2 + b_3 + b_4}{a_1 + a_2 + a_3 + a_4} - 1 = \frac{kR}{Bl K_p} \tag{7.8}$$

The actuator function is then integrated into the hybrid system as a discrete transfer function, as illustrated in Figure 7.2.



Figure 7.2: Hybrid model including discretized actuator model

7.3.2. Hybrid test for nonlinear structure with FFr

For the FF_r, impedance control requires an adjustment of the proportional gain to changing structure stiffness. For a nonlinear stiffness this means, that for every time step the proportional and feedforward gains K_p and ff_r are recalculated. The structure stiffness is modeled as a function of the applied displacement and highly nonlinear. The more the structure is deformed, the more it stiffens up, such that $K_{sub} = k_1(1+2|x|)$, where x is the actuator displacement. In response to the computed gains, the parameters a_i and b_i in equation (7.6) are then computed dynamically, too. The discrete transfer function block can therefore not be applied anymore. Instead, the actuator response is also modeled numerically with an Sfunction, which replaces the discrete transfer function block in Figure 7.2. In order to solve for the actuator displacement numerically as a function of the previous displacements and varying parameters, the transfer function is rearranged.

$$H_{vx,cl}(z) = \frac{X(z)}{R(z)} = \frac{a_1 z^3 + a_2 z^2 + a_3 z + a_4}{b_1 z^3 + b_2 z^2 + b_3 z + b_4} = \frac{a_1 + a_2 z^{-1} + a_3 z^{-2} + a_4 z^{-3}}{b_1 + b_2 z^{-1} + b_3 z^{-2} + b_4 z^{-3}}$$
(0.1)
$$(b_1 + b_2 z^{-1} + b_3 z^{-2} + b_4 z^{-3}) X(z) = (a_1 + a_2 z^{-1} + a_3 z^{-2} + a_4 z^{-3}) R(z)$$

Applying the backward-difference discretization [19], the new displacement can be computed out of the three previously applied and commanded displacements and the actual commanded displacement (reference input).

$$b_{1}x(kT) + b_{2}x[(k-1)T] + b_{3}x[(k-2)T] + b_{4}x[(k-3)T] = a_{1}r(kT) + a_{2}r[(k-1)T] + a_{3}r[(k-2)T] + a_{4}r[(k-3)T]$$

$$x(kT) + \frac{b_{2}}{b_{1}}x[(k-1)T] + \frac{b_{3}}{b_{1}}x[(k-2)T] + \frac{b_{4}}{b_{1}}x[(k-3)T] = \frac{a_{1}}{b_{1}}r(kT) + \frac{a_{2}}{b_{1}}r[(k-1)T] + \frac{a_{3}}{b_{1}}r[(k-2)T] + \frac{a_{4}}{b_{1}}r[(k-3)T]$$

$$x(kT) = \frac{a_{1}}{b_{1}}r(kT) + \frac{a_{2}}{b_{1}}r[(k-1)T] + \frac{a_{3}}{b_{1}}r[(k-2)T] + \frac{a_{4}}{b_{1}}r[(k-2)T] + \frac{a_{4}}{b_{1}}r[(k-3)T]$$

$$(0.2)$$

In the following test, the initial gain is designed adequately for the initial stiffness k_1 , by applying 80% of the allowable limit.

$$K_{p,k1} = 0.8K_{p\max,k1} = 0.8\frac{(Bl)^2(cL+mR) + c(k_1L^2 + R(cL+mR))}{Bl\,Lm}$$
(0.3)

The gain will be adjusted to the nonlinear stiffness so that the system impedance in equation (7.12) remains constant in DC.

$$\hat{K} = \left| \frac{F_{ext}}{x_{ext}} \right| = \frac{Lms^2 + (mR + cL)s + (cR + Bl^2)}{R + Ls}s + \frac{Bl K_p}{R + Ls} + k$$
(0.4)

If the structure stiffness is highly increased, then the impedance cannot be maintained. As explained in the earlier chapter, the FF_r is not adequate anymore once

the structure stiffness gets too high. If the structure stiffness remains below the critical limit, then the stiffness of the coupled system can be maintained by lowering the gain as in equation (7.13).

$$K_{p} = K_{p1} - \frac{R}{Bl}(k - k_{1})$$
(7.13)

Figure 7.3 simulates the nonlinear test for a sine wave ground acceleration, plotting the structure stiffness, proportional gain and feedforward gain as a function of the applied displacement.



Figure 7.3: FF_r for nonlinear stiffness

Good actuator tracking is achieved by the continuous adjusting of the proportional gain and the feedforward gain. The simulation illustrates how accurate tracking and impedance control can be combined by using the FF_r. Again, however, this requires that the nonlinear structure stiffness is known well and does not exceed certain limits.



Figure 7.4: Hybrid model including FF_x

The model in Figure 7.4 models the response for the FF_x . As explained earlier, impedance control takes place due to the external feedforward loop even though the gain is kept constant. Again, however, the structure stiffness must be known in order to model the correct feedforward input. This is not necessary if the force derived feedforward is applied. As the FF_x and FF_{if} are equivalent for linear structure stiffness, the model in Figure 7.4 is also used as a comparison to the laboratory tests using force derived feedforward.

7.4. Labview real-time model

Finally, the laboratory tests are performed using the real-time target machine. Figure 7.5 relates the functionalities of the hybrid test setup to the corresponding components in the simulink model. A computational time step of *1 ms* is chosen. In order to guarantee a smooth computation, only time critical computations are performed on the real-time target machine. The output signals are sent over the network to the host machine, where the non time critical actions, such as monitoring and processing of the output, are performed.



Figure 7.5: Deployment diagram for hybrid simulation



Figure 7.6: Block diagram of hybrid tests

Figure 7.6 represents the complete block diagram for the hybrid test. The top loop is time critical and performs every millisecond. It reads from and writes to the physical

device, processes the signal input and computational model, and writes all the output to a buffer. The bottom loop is not time critical and performs only as long as the time critical loop performs perfectly. It reads the data from the buffer, processes it and sends it over the network to the host machine. Figure 7.7 shows the tasks which are then performed on the host machine. The information is received over the network and is then processed, monitored and written to an output file.



Figure 7.7: Block diagram on the host machine

7.5. Test results

The following illustrates the effect of the actuator delay both in the time and frequency domain. The first paragraph shows the simulated and modeled time response of four different examples. The upper and lower allowable gains are computed, simulated and measured for a chosen low frequency ground acceleration. Then, the effect of the actuator delay on a hybrid test is presented with the FRF over the whole actuator bandwidth. Each time the three masses, dampers and springs are chosen as identical. Therefore $k = k_1 = k_2 = k_3$ must match the stiffness of the rubber

bands and the corresponding brass bar. The masses and dampers are chosen reasonably as a function of the stiffness.

7.5.1. Experiment 1

In the first tests the upper and lower limits for the proportional gain are computed, simulated and measured. All test results are performed for a ground acceleration of a *1 Hz* sine wave and represent the gain limits for the uncompensated scheme and all feedforward schemes. The actuator displacements are kept low in order to approximate a linear structure stiffness. Nevertheless, this approximation is one possible reason, why the measured and simulated results sometimes vary. The maximum gains for the reference and displacement derived schemes are repeated below.

$$K_{pffr} < \frac{Bl R}{L \operatorname{conv} G_{v}} + c \frac{Bl^{2}L + kL^{2} + mR^{2} + LRc}{Bl \operatorname{Lm} \operatorname{conv} G_{v}}$$

$$K_{pffx} \leq \frac{(cL + mR)(cR + Bl^{2} + kL)}{Bl \operatorname{Lm} \operatorname{conv} G_{v}}$$
(7.14)

They are also verified in the root locus plots, which are presented for every example. Note that the root locus assumes a continuous numerical substructure and a linear physical substructure. The simulated gain limits result from trial and error from the simulink models. In comparison to the calculated limits of the continuous system, these block diagrams solve for the limits in the discretized system. Those approximations are then compared with the measured results.





FF_x	$K_{p\min}$	0.60	0.65	0.5			
	$K_{p \max}$	1.58	1.65	2.0			
FF _{if}	$K_{p\min}$	0.60	0.4	0.8			
	$K_{p \max}$	1.58	1.5	1.5			
$m = 0.005 lb \frac{s^2}{in}$ $k = 6.7 \frac{lb}{in}$ $c = 0.1 \frac{lb s}{in}$							
RL Kp for NoFF							
NoFF	K _{n min}	1.0	1.0	0.9			
	$K_{p \max}$	1.5	1.5	1.8			
RL Kp for FFr $RL Kp for FFr$ $Kpmax=1.5$ $Kpmin=1.05$ $Kpmin=1.05$ $Kpmin=1.05$ $Kpmin=1.05$ $Kpmin=1.05$ $Real Axis$ $Real Axis$							
FF _r	$K_{p\min}$	1.03	1.03	0.9			
	$K_{p \max}$	1.5	1.5	1.8			



FFr	$K_{p\min}$	0.34	0.35	0.3				
	$K_{p \max}$	1.42	1.6	2.0				
	RL Kp for FFx 80 60 40 40 40 40 40 40 40 40 40 4							
FF _x	$K_{p\min}$	0.76	0.8	0.7				
	$K_{p \max}$	1.86	2	2.4				
FF _{if}	$K_{p\min}$	0.76	0.8	0.6				
	$K_{p \max}$	1.86	2	1.7				

Table 7-1: Computed, simulated and measured stability bounds for the proportional gain

Summarizing the table, the simulated results are very close for the continuous and discrete model. Despite of the structure's nonlinearity they are also relatively close to the measurements. The maximum gain in the measurement is usually a bit higher for the sine wave, the lower bound for the gain could be verified with better accuracy. With the load cell, instability occurs quicker both for lower and higher gains. This is reasonable as it is very noisy and captures the structure nonlinearity.

The above hybrid models were all sufficiently damped to achieve a stable test for the allowable range of the proportional gain. In reality however, this damping might be significantly lower. Moreover the tests above were only applied for one single frequency. In the following, the effect of the actuator delay will be examined over the whole actuator bandwidth.

7.5.2. Experiment 2

In order to show the effect of the actuator delay over the whole frequency range, the model in Figure 7.1 is tested for three equivalent spring-mass-dashpot systems with $m = 0.02 lb s^2 / in$ and k = 23 lb / in. The eigenfrequencies of the model are all within the actuator bandwidth, i.e. $f_1 = 2.4 Hz$, $f_2 = 6.7 Hz$ and $f_3 = 9.7 Hz$. In the first test a damping of c = 0.6 lb s / in is applied. This corresponds to $\zeta = 20\%$ damping ratio of the first eigenmode. Figure 7.8 models the actuator displacement in response to the multi-sine input representing the normalized ground acceleration.



Figure 7.8: Increased resonance peak due to actuator delay

In comparison to the simulated response, where perfect actuator tracking is assumed, the other models show an increased resonance peak which results from the negative damping due to the actuator delay. The FF_x and FF_{if} show an even higher peak than the FF_r due to the softer and slower actuator. Models with lower damping therefore need compensation for the actuator delay. In the following the I-modification scheme

[10] will be applied to stabilize a low damped system with c = 0.1 lb s / in, which yields $\zeta_1 = 3\%$ damping ratio of the first eigenmode, and $\zeta_2 = 19\%$ and $\zeta_3 = 13\%$ for the second and third eigenmode.

7.5.3. Experiment 3



7.5.3.1. Hybrid simulation algorithm with I-modification

Figure 7.9: Hybrid loop with I-modification scheme

The hybrid simulation algorithm is shown schematically in Figure 7.9. Of particular note in this figure is the I-modification scheme. The I-modification alters the force feedback to the numerical substructure by comparing the commanded displacement r, and applied displacement x and multiplies this displacement error with an estimate of the initial structure stiffness, K_I .

$$\Delta F = K_{I}(r - x) \tag{7.15}$$

Ideally, if the I-modification used the exact structure stiffness, then the actuator delay would be completely compensated for. However, since the structure could be nonlinear, and its stiffness could vary during the simulation, K_I cannot exactly equal

the structure stiffness. Therefore, an over- or under-compensation of the actuator delay could occur. Both cases can run the hybrid simulation unstable. Therefore, a procedure for the analysis of the stability of the hybrid simulation algorithm with Imodification is discussed.

7.5.3.2. Stability analysis of the I-modification scheme

The stability analysis is carried out by linearizing the physical substructure model, and using the model of the actuator described in section 3. The proportional gain K_p is designed for good tracking in free motion. Considering the entire hybrid simulation system as a cascaded close-loop system, from Figure 7.9, the transfer function for the entire hybrid system can be obtained. Figure 7.10 shows the root locus plot for this transfer function for increasing I-modification stiffness.



Figure 7.10: Root locus for I-modification stiffness

The plot shows that for the low damping of the hybrid model, I-modification is necessary in order to shift the unstable poles into the left half plane. However there is also an upper limit for the I-modification stiffness, when a complex conjugate pair of poles shifts into the right half plane. The lower and upper limits are shown in Figure 7.10 and were also confirmed experimentally. For non-realtime simulation, where the actuator dynamics need not be considered, it has been remarked by Combescure and Pegon [10] that using an I-modification that is more positive than the actual stiffness guarantees stability. However, the above analysis shows that for realtime simulation, where actuator dynamics are significant, there is also an upper bound for the I-modification. This should be considered when designing a hybrid simulation controller.

In the following hybrid simulation, the Newmark's constant average acceleration algorithm is used for the numerical substructure, and an I-modification factor of $K_1 = 28$ lb/in = 4900 N/m is applied, which is within the stability bounds established above. First, a multisine with randomized phase is used as the ground acceleration input, and the frequency response from the ground acceleration to the displacement of the first mass is measured. As shown in Figure 7.11 and Figure 7.12, the frequency response obtained from the hybrid simulation agrees closely with that obtained from a purely numerical solution. The discrepancy is somewhat pronounced at the third resonance, where the measurement is noise dominated. This illustrates that feedforward in combination with the I-modification scheme allows for stable and accurate hybrid testing even for structures of very high and nonlinear stiffness.



Figure 7.11: FRF from ground acceleration to actuator displacement



Figure 7.12: FRF in logarithmic scale

Next, the hybrid simulation is carried out with the El Centro 1940 earthquake acceleration record as the input. Figure 7.13 shows the reference and applied displacement of the actuator during the hybrid simulation. Again it is seen that good tracking is achieved, even though a high nonlinear structure stiffness is applied and the gain is maintained, which has been originally designed for the actuator in free motion.



Figure 7.13: Good tracking with forced derived feedforward

7.6. Summary

Stability in simple closed loop testing is achieved even though a phase lag is present. Hybrid testing introduces a second stability bound for the proportional gain, as actuator delay leads to negative damping and eventually instability in the hybrid model. The force derived feedforward scheme in combination with the Imodification scheme is finally shown as a relatively simple but very powerful tool to run hybrid tests both accurately and stable, even though the structure stiffness is not known well or nonlinear. As a general design rule for structures of nonlinear or unknown stiffness, the following three steps are recommended.

1. Design the proportional gain for the lowest known structure stiffness, or for free motion if the stiffness is completely unknown.

- 2. Apply the FF_{if} .
- 3. Apply the I-modification scheme if the structure stiffness can be estimated well.

By designing the actuator gain for the lowest known stiffness, actuator instability is impossible to happen. The I-modification scheme will increase the accuracy of the hybrid test and compensate for the negative damping due to the actuator delay. Depending on the hybrid structure however, overcompensation might also lead to instability. Its application should therefore not be applied if the structure stiffness is completely unknown.

8. CONCLUSION

Accurate and stable actuator tracking is only possible if the actuator-structureinteraction is integrated into the control design. This is possible by controlling the actuator impedance, which requires that the actuator deforms in response to the actuator-structure-interaction force. In order to maintain the desired impedance of the coupled actuator-structure system, the actuator impedance must adjust to a varying structure stiffness.

Feedforward control has been shown as a tool to compensate for actuator-structureinteraction while maintaining the required actuator impedance. The FF_{if} is applicable even for nonlinear or unknown structure stiffness as it automatically adjusts the actuator impedance to a varying structure stiffness.

Finally, feedforward was applied to hybrid testing. This introduced a second stability bound, as actuator delay has the effect of negative damping on hybrid system and eventually leads to instability. The I-modification scheme has been shown as a possible tool to compensate for the actuator delay. As a general design rule for structures of nonlinear or unknown stiffness, the following three steps are recommended.

1. Design the proportional gain for the lowest known structure stiffness, or for free motion if the stiffness is completely unknown.

2. Apply the force derived feedforward.

3. Apply the I-modification scheme if the structure stiffness can be estimated well.

The feedforward input was designed to compensate for the actuator-structureinteraction statically. With this approach the system impedance could be controlled and stability guaranteed. In regard to accurate actuator tracking however, the FF_{if} only compensates well for force disturbances in the low frequency range, but not in the range of the actuator resonance frequency. In addition to that, all shown feedforward schemes require a close knowledge of the actuator properties.

As a further research the aim should be, to build an actuator-control system which adjusts the gain and feedforward according to unknown system parameters both in regard to the structure stiffness and the actuator properties. Several of those control schemes have been applied in robotic systems and are presented in appendix B. In particular the learning impedance control and model based prediction are able to converge the control system towards a target impedance by continuously estimating and adjusting system parameters. Those models are mostly insensitive to system uncertainties but require an extensive experience in control system analysis. A full compensation of the actuator-structure-interaction by feedforward however will never be possible without the prediction of the interaction force. Different estimation and adapting schemes could further compensate for the interaction.

Summarizing, the stability problem of the actuator due to stiff structures could be solved. The force derived feedforward scheme is able to track a transient input stable and accurately even for structures of high, nonlinear and unknown stiffness. The improved actuator tracking will increase accuracy and stability also for hybrid testing.

A.APPENDIX – Algorithms in hybrid testing

Chapter 2 briefly reviewed the Newmark algorithms which are preferably used in hybrid testing. This appendix shows the properties of all Newmark schemes as a response to varying system parameters. Other algorithms have been developed which improve accuracy and stability of the system and will be presented in detail as well.

A.1. Newmark method



Figure A 1: Newmark methods

Figure A 1 shows again the Newmark family methods. The tuning of the parameters α , β and γ creates both explicit and implicit methods with different characteristics, which will be discussed in more detail now.

A.1.1. Newmark β method

$$\dot{u}_{n+1} = \dot{u}_n + (1 - \gamma)\ddot{u}_n h + \gamma \ddot{u}_{n+1} h$$

$$u_{n+1} = u_n + \dot{u}_n h + (\frac{1}{2} - \beta)\ddot{u}_n h^2 + \beta \ddot{u}_{n+1} h^2$$

$$M \ddot{u}_{n+1} + C \dot{u}_{n+1} + K u_{n+1} = P_{n+1}$$
(A1)

In the Newmark β method one eigenvalue is always zero, the other two eigenvalues are a complex conjugate pair. Choosing the parameters β and γ correspondingly, the Newmark β method shows different characteristics. Predefined combinations of the parameters form known explicit and implicit methods, such as the central difference method, the constant acceleration method, the constant average acceleration method and the linear acceleration method.

A.1.2. HHT method

$$\dot{u}_{n+1} = \dot{u}_n + (1-\gamma)\ddot{u}_n h + \gamma \ddot{u}_{n+1} h$$

$$u_{n+1} = u_n + \dot{u}_n h + (\frac{1}{2} - \beta)\ddot{u}_n h^2 + \beta \ddot{u}_{n+1} h^2$$

$$M\ddot{u}_{n+1} + (1+\alpha)C\dot{u}_{n+1} - \alpha C\dot{u}_n + (1+\alpha)Ku_{n+1} - \alpha Ku_n = (1+\alpha)P_{n+1} - \alpha P_n$$
(A2)

The Hilber-Hughes-Taylor method [16], or α -dissipation method, includes the additional weighting factor α . This allows further "tuning" for stability and numerical damping. For nonlinear systems [12], the generalized α -method is endowed with stability in an energy sense and guarantees energy decay in the high-frequency range as well as asymptotic annihilation. However, overshoot and heavy energy oscillations in the intermediate-frequency range were exhibited.

A.1.3. Explicit methods

Explicit methods compute the response of the structure of step i+1 based on the results of step *i*. Explicit methods are easier to implement and usually preferred for hybrid simulations, however they usually have more restrictive criteria related to the natural step ωh .

A.1.3.1 Central difference method

The central difference method derives from the Newmark β method for $\beta = 0$ and $\gamma = 0.5$. $\beta = 0$ means that the displacements are calculated explicitly.

$$\dot{u}_n = \frac{u_{n+1} - u_{n-1}}{2h} \quad \ddot{u}_n = \frac{u_{n+1} - 2u_n + u_{n-1}}{h^2}$$
(A3)

The method becomes unstable for $\Omega = \omega h > 2$ independently of the natural damping and has a growing negative period distortion for growing values of the natural damping and step size.

A.1.3.2 Explicit α method

$$\dot{u}_{n+1} = \dot{u}_n + \frac{h}{2}(\ddot{u}_n + \ddot{u}_{n+1})$$

$$u_{n+1} = u_n + \dot{u}_n h + \frac{1}{2}\ddot{u}_n h^2$$

$$M\ddot{u}_{n+1} + (1+\alpha)C\dot{u}_{n+1} - \alpha C\dot{u}_n + (1+\alpha)Ku_{n+1} - \alpha Ku_n = (1+\alpha)P_{n+1} - \alpha P_n$$

$$\beta = 0 \quad \gamma = \frac{1}{2} \quad \alpha \ge 0$$
(A4)

The explicit α method results from the HHT method for the given parameter values. Positive values for α are allowed if β is set to zero. This way the method becomes explicit and conditionally stable. For $\alpha = 0$ the method reduces to the central difference method with a stability limit of $\Omega = 2$. However it is a one-step method, while the central difference method is a two-step method, although they are mathematically identical [50]. For positive α this limit decreases and stability becomes more critical. The numerical damping depends highly on the natural step, which was not the case for the other methods.

A.1.3.3 Modified Newmark Method

In the modified Newmark method an additional parameter ρ is added to the explicit α -method.

$$\dot{u}_{n+1} = \dot{u}_n + \frac{h}{2}(\ddot{u}_n + \ddot{u}_{n+1})$$

$$u_{n+1} = u_n + \dot{u}_n h + \frac{1}{2}\ddot{u}_n h^2$$

$$M\ddot{u}_{n+1} + [(1+\alpha)K + \frac{\rho}{\Delta t^2}M]u_{n+1} = P_{n+1} + (\alpha K + \frac{\rho}{\Delta t^2}M)u_n$$

$$\rho \le 0 \quad \alpha \ge 0$$
(A5)

With a careful selection of the parameters the method provides numerical damping. However, stability is only provided for an interval of ωh as defined in equation (A6)

•

$$\sqrt{-\frac{\rho}{\alpha}} \le \omega h \le \frac{1 + \sqrt{1 - (1 + \alpha)\rho}}{1 + \alpha}$$
(A6)

While the magnitude of stiffness-proportional damping is proportional to the natural frequency of a system, that of mass-proportional damping is inversely proportional to the frequency. Thus, in order to provide for small damping for the fundamental mode and large damping for the higher modes, it is best to have a positive α and a negative ρ [51].

For $\alpha = \rho = 0$ the central difference method, the explicit Newmark method and the modified Newmark method are all mathematically equivalent to each other. This means that they have identical stability and accuracy properties. However, their numerical characteristics can differ. For instance the central difference method has undesired significant error-propagation effects, which is less problematic for the other two methods [50].

A.1.3.4 Constant acceleration method

The constant acceleration method results for $\beta = 0$ and $\gamma = 0$.

$$\dot{u}_{n+1} = \dot{u}_n + \dot{u}_n h$$

$$u_{n+1} = u_n + \dot{u}_n h + \frac{1}{2} \ddot{u}_n h^2$$

$$M \ddot{u}_{n+1} + C \dot{u}_{n+1} + K u_{n+1} = P_{n+1}$$
(A7)

This means that both the displacement and the velocity can be calculated explicitly, however with the price of stability problems. Both high values of the natural step and low values of the natural damping lead to instability. Unlike most other methods, the constant acceleration method has an initial negative numerical damping for zero natural damping.

A.1.3.5 Further explicit methods

Chang [6] presented an unconditionally stable explicit algorithm in 2002. He used different weight factors as in the explicit Newmark method. The method comprises the advantages of explicit schemes and provides for unconditional stability simultaneously. The amplification matrix is the same as for the constant average acceleration method and therefore shows the same properties of unconditional stability and energy conservation. Moreover, it provides better error propagation than comparable algorithms, such as the Newmark explicit method.

Improved numerical dissipation for explicit methods is also achieved by applying quadratic functions for the parameters α , β and γ [5]. Generally, numerical dissipation can be introduced into the explicit form of the Newmark β method choosing $\gamma = 1/2$. This however brings the problem that the lower modes are damped too much, as the numerical damping is approximately proportional to the step ($\overline{\zeta} \sim \overline{\Omega}$). Implementing α , β and γ as functions of the inverse of the mass and stiffness matrices of the structural system and the size of the integration time step, a quadratic relationship between the numerical damping and the step becomes possible. This means that particularly the higher frequencies are damped, which are usually due to noise, while no undesired damping for the low frequencies takes place.

A.1.1. Implicit methods

Implicit methods require information about the structural response at the displacement target in order to satisfy equilibrium at the end of the step. They provide for better stability characteristics and enable the use of bigger time steps.

A.1.1.1 Constant average acceleration method

The constant average acceleration method derives again from the Newmark β method but is implicit and unconditionally stable. The period distortion is positive (numerical period is longer than natural) for low natural damping and negative for high natural damping. The method has been used for the hybrid tests in this study as it is stable and does not introduce any numerical damping.

$$\gamma = \frac{1}{2} \quad \beta = \frac{1}{4}$$

$$\dot{u}_{n+1} = \dot{u}_n + \frac{1}{2}h(\ddot{u}_n + \ddot{u}_{n+1})$$

$$u_{n+1} = u_n + \dot{u}_n h + \frac{1}{4}h^2(\ddot{u}_n + \ddot{u}_{n+1})$$

$$M\ddot{u}_{n+1} + C\dot{u}_{n+1} + Ku_{n+1} = P_{n+1}$$
(1)

A.1.1.2 Linear acceleration method

$$\gamma = \frac{1}{2} \quad \beta = \frac{1}{6}$$

$$\dot{u}_{n+1} = \dot{u}_n + \frac{1}{2}h(\ddot{u}_n + \ddot{u}_{n+1})$$

$$u_{n+1} = u_n + \dot{u}_n h + \frac{1}{3}\ddot{u}_n h^2 + \frac{1}{6}\ddot{u}_{n+1}h^2$$

$$M\ddot{u}_{n+1} + C\dot{u}_{n+1} + Ku_{n+1} = P_{n+1}$$
(2)

The linear acceleration method is another implicit and unconditionally stable method and can also include both positive and negative period distortion.

A.1.1.3 Newmark α Method (Hughes)

This method again derives from the HHT method shown in equation (A2), however another combination of the parameters is applied. If α is in the shown negative range and the other parameters are computed correspondingly, then the method is implicit and unconditionally stable. For $\alpha = 0$ the method reduces to the constant average acceleration method (A8). For negative α , the implicit part is weighted more and leads to more numerical damping.

$$-\frac{1}{3} \le \alpha \le 0 \quad \gamma = (\frac{1}{2} - \alpha) \quad \beta = \frac{(1 - \alpha)^2}{4}$$

$$\dot{u}_{n+1} = \dot{u}_n + h[(\frac{1}{2} + \alpha)\ddot{u}_n + (\frac{1}{2} - \alpha)\ddot{u}_{n+1}]$$

$$u_{n+1} = u_n + \dot{u}_n h + \frac{h^2}{4}[(\alpha + 1)^2\ddot{u}_n + (\alpha - 1)^2\ddot{u}_{n+1}]$$

$$M\ddot{u}_{n+1} + (1 + \alpha)C\dot{u}_{n+1} - \alpha C\dot{u}_n + (1 + \alpha)Ku_{n+1} - \alpha Ku_n = (1 + \alpha)P_{n+1} - \alpha P_n$$

(3)

An important difference to the earlier discussed β -method is the third eigenvalue. In addition to the complex conjugate pair, there is a real eigenvalue for any $\alpha \neq 0$. While for negative α the real eigenvalue is small and not of major interest in respect to stability, for positive α especially this real eigenvalue limits the stability as shown above in the explicit α method (A4). Shing [53] showed that convergence can be insured even though the actual stiffness matrix of a structure becomes non-positive definite as long as there is sufficient mass and damping in the structure and the time integration integral is sufficiently small. He also showed that the implicit α method always implies positive numerical damping and therefore stability. In the case of unloading and reloading of a nonlinear system, the unconditional stability of an implicit scheme depends on the time step h [31].

A.1.4.4 Operator-Partitioning Implementation

The operator-partitioning algorithm is a combined implicit-explicit integration algorithm. The following predictors are calculated explicitly:

$$\hat{\dot{u}}_{n+1} = \dot{u}_n + (1 - \gamma) \ddot{u}_n h$$

$$\hat{u}_{n+1} = u_n + \dot{u}_n h + (\frac{1}{2} - \beta) \ddot{u}_n h^2$$

$$M \ddot{u}_{n+1} + C \dot{u}_{n+1} + K u_{n+1} = P_{n+1}$$
(A11)

The predicted displacements are applied to the physical model and the force response is measured. Then one can solve for the new acceleration and then update with correctors.

$$\begin{aligned} \ddot{u}_{n+1} &= \tilde{M}^{-1} \tilde{P}_{n+1} \\ \dot{u}_{n+1} &= \hat{\dot{u}}_{n+1} + \gamma \ddot{u}_{n+1} h \\ u_{n+1} &= \hat{u}_{n+1} + \beta \ddot{u}_{n+1} h^2 \end{aligned}$$
(A12)

The advantage of using the operator splitting method is that unconditional stability is guaranteed for non-linear structures of the softening type. In this study the operator splitting method has been used to compute the next displacement. However only one iteration based on the initial stiffness has been applied.

Combescure [10] used a linearization of the earlier shown α -method, the α -Operator Splitting. By using a predictor step, this implicit scheme becomes non-iterative. Neglecting the actuator dynamics, i.e. by assuming that the actuator applied the predictor displacement accurately, the system is unconditionally stable. The feedback force is approximated by

$$r_{n+1}(d_{n+1}) \approx K^{I} d_{n+1} + (\tilde{r}_{n+1}(\tilde{d}_{n+1}) - K^{I} \tilde{d}_{n+1})$$
(A13)

where d_{n+1} and \tilde{d}_{n+1} are the effectively applied and predicted displacements and K^{1} is the initial stiffness. They found out that the α -Operator Splitting method is a good alternative to the more complex iterative schemes.

A.2. Further integration methods

A.2.1. Higher order accuracy integration method

The common trapezoidal rule used for integration is of second order accuracy. In general, methods of α^{th} order approximation are 2α -order accurate [46]. Therefore, there exists the possibility to improve the accuracy of the numerical algorithm by taking higher order polynomial functions.

A.2.2. Solving the integrated equation of motion

Solving the integrated equation of motion serves as another effective tool to increase accuracy and decrease error propagation [7].

$$m\dot{u} + cu + \overline{r} = \overline{f}$$

$$m\dot{u}(0) + cu(0) + \overline{r}(0) = \overline{f}(0)$$
(A14)

The initial conditions must be satisfied like shown in equation (A14), the rest of the procedure is comparable to the original. An integration of the external forces and the restoring forces smoothes out the jagged character of the dynamic loading, eliminates the adverse linearization errors, captures better the external forces and thus leads to better accuracy.

Depending on the chosen algorithm, the method can turn out as conditionally stable and display numerical damping [7]. By integrating the central difference method, the displacement cannot simply be computed explicitly anymore as it evolves from the implicitly computed velocity in the original scheme. To avoid iterations and undesirable unloading or overshoot, the restoring force for the new time step can be estimated as in equation (A15) where k is the initial stiffness of the system.

$$\int r_{n+1}dt = \int r_n dt + hkd_n + \frac{1}{2}kh^2v_n = \int r_n dt + hr_n + \frac{1}{2}kh^2v_n$$
(A15)

After estimating the restoring force, first the velocity and then the displacement for the next time step are computed. This method becomes more effective if the restoring force is estimated with an additional implicit term [2], such as represented in equation (A16).

$$\int r_{n+1}dt = \int r_n dt + hr_n + \frac{1}{2}h^2(v_n + v_{n+1})$$
(A16)

This estimate results out of the assumption of linear stiffness, so that the change in the restoring force integral yields equation (A17).

$$\Delta \int r_{n+1} dt = \int r_{n+1} dt - \int r_n dt = \frac{1}{2} hk(d_n + d_{n+1})$$
(A17)

This successfully combines advantages of the integral form in handling rapidly varying loads and stiffness degradation with the unlimited step size associated with implicit methods.

A.2.3. State-space

Wang [59] used an integration of the state-space procedure with Nakashima's operator-splitting concept. The state space procedure is based on an interpolation of the discrete excitation signals for piecewise convolution integrals. The Operator-Splitting State-Space Procedure method turns out as conditionally stable. However it
exhibits more accurate results as the Newmark method as it provides for perfect conservation of the natural frequency and damping. Hence it does not lead to any period distortion or any energy dissipation and is therefore recommended for pseudodynamic testing if highly accurate results are desired.

Krenk [24] developed a state-space time integration with fourth-order accuracy and energy control for linear systems. The algorithm is derived by integrating the phasespace representation and evaluating the resulting displacement and velocity integrals via integration by parts. Krenk compared his method to common integrals, such as the generalized α -method, and showed that the accuracy increases from second- to fourth-order by evaluating the integrals via integration by parts.

A.2.4. Modal weighting technique

The modal weighting technique [15] is a hybrid method that spans both the Cartesian and modal bases simultaneously. By introducing a change of basis to modal generalized coordinates, the system can be expressed by a number of uncoupled second-order equations. An undamped, elastic, second-order system can be expressed that way as:

$$X(t) = \sum_{m=1}^{n_{eq}} x_m(t) \Psi_m$$
 (A18)

where x_m is the scalar modal expression of the m^{th} mode shape Ψ_m , which is obtained from

$$x_m = \Psi_m^T M X(t) \tag{A19}$$

So, the equation of motion could be decomposed in n decoupled equations:

$$\sum_{m=1}^{n_{eq}} (\ddot{x}_m \Psi_l^T M \Psi_m + x_m \Psi_l^T K \Psi_m) = 0$$
(A20)

which can be further simplified to

$$\ddot{x}_l + \lambda_l x_l = 0 \tag{A21}$$

To make the system stable, the natural frequencies are multiplied by a parameter γ with $0 < \gamma < 1$. This way the higher modes were reduced or completely eliminated. So, the eigenmatrix is multiplied by the modal norm $\Psi \Gamma \Psi^T$ with

$$\Gamma = \begin{pmatrix} \gamma_1^2 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \gamma_n^2 \end{pmatrix}$$
(A22)

According to the stability conditions of the amplification matrix the spurious highfrequency oscillation modes can be attenuated.

B.APPENDIX – Impedance design methods in robotics

This appendix reviews different applied impedance schemes in robotics. It shows different ways to control the actuator impedance for both cases where all the system parameters are known and cases where the structure stiffness or even more parameters are unknown.

B.1. Defining target impedance

A rule of thumb for choosing an adequate target impedance should be to make the manipulator impedance proportional to the structural compliance. According to optimization theory the impedance is chosen when the performance index, which is a function of interaction forces and displacements, is at a maximum or minimum.

The objective function to be minimized is:

$$Q = \int_{0}^{\infty} \{ (F / F_{tol})^{2} + [(r - x) / x_{tol}]^{2} \} dt$$
(B1)

Where x_{tol} and F_{tol} are the position and force tolerance, r and x are the commanded and applied actuator displacements. The optimal damping factor ζ_{opt} results from the optimal stiffness $k_{opt} = F_{tol} / x_{tol}$ as $\zeta_{opt} = \sqrt{2(k_{opt}m)}$. Even with extremely little information about the structure, the interaction between actuator and structure may be controlled so as to meet task specifications.

B.2. Design models for known structure stiffness

If a target impedance is found, then the desired interaction force can be derived. If this force is not achieved, then the model has an impedance different from the target impedance. Dividing the force by the target impedance yields the desired displacement error. The difference between the desired and actual displacement error then serves as the adjusting feedforward input as shown below in Figure B 1.



Figure B 1: Feedforward with target impedance

B.3. Design models for unknown stiffness and/or parameter uncertainties

If the structure stiffness is not known or varies, then adaptive schemes have to be developed in order to achieve the target impedance. In the following different adaptive schemes will be presented briefly.

B.3.1. Adaptive methods

Seraji and Colbaugh [45] showed two schemes generating the reference position trajectory required to produce a desired contact force despite lack of knowledge of the structure stiffness and location. The direct adaptive control scheme by [45] is represented in Figure B 2. In this case not the displacement but the force error is used to create the corresponding reference input. In contrast to the previously shown scheme, the force error basically passes a PID controller before it is fed back to the system. This allows a better and especially adjustable impedance design.



Figure B 2: Direct adaptive control [45]

Jung, Hsia and Bonitz [22] showed a new, simple and stable force tracking impedance control scheme that tracks a specified desired force and compensates for uncertainties in environment location and stiffness as well as actuator dynamics. They defined for an accurate force control the following three aims:

- 1. A position tracking error due to unknown actuator dynamic uncertainty should be minimized.
- 2. A desired force should be directly commanded to have the force tracking capability.
- 3. The controller must be robust enough to deal with unknown environment stiffness and position.

Their main idea was to minimize the force error directly by using a simple adaptive gain when tracking unknown structure stiffness.

Carelli and Kelly [4] presented a model, where parameters estimation combined with an additional compensation through an extra signal achieved asymptotic tracking properties of the adaptive controller.

B.3.2. Estimator model

Many adaptive methods include the estimation of parameters. In the indirect adaptive strategy by Seraji and Colbaugh [45], the force error is fed back into the system to adjust the reference input such like in the earlier example. Additionally, the measured force and displacement are used to estimate structural parameters online. The required reference position is then computed based on these estimates.

Shibata [47] obtained a robust impedance controller using a disturbance observer. The dynamic characteristics of the object are well estimated by an adaptive identification algorithm. The disturbance observer eliminates the modeling errors of the system and increases the robustness of the system. This made it possible to estimate the accurate parameters of the dynamic model and the structure stiffness.

B.3.3. Learning impedance

Because of the nonlinearity of the dynamics, the identification and estimation techniques cannot always be deployed easily. The ideal control system is supposed to adapt to any changing parameters as well. In other words, the model needs not only be able to adjust to unknown parameters, but it has to be able to adjust to changing parameters as well. Learning controllers are referred to the class of control systems that generate a control action in an iterative manner to execute a prescribed action. The reference input is computed in a more complex way than in the earlier presented strategies. However it allows for the computation of a control mechanism, which passes all the requirements for hybrid testing and dynamic testing in general. This means a stable impedance control mechanism which successfully handles unknown or changing structure stiffness or model parameters.

Cheah and Danwei [8] developed a learning impedance control. The feedforward control inputs are learned such that the system tracks the desired motion and force trajectories as the actions are repeated. First, the stability limits for their control scheme were derived and the target impedance defined. Then, with the commanded and desired reference displacement r(t), the measured actual displacement x(t) and the measured actual force $F_k(t)$, the equation of motion is solved for the force error $\omega_k(t)$ as shown in equation (B2).

$$\omega_{k}(t) = M_{m}[\ddot{r}(t) - \ddot{x}_{k}(t)] + C_{m}[\dot{r}(t) - \dot{x}_{k}(t)] + K_{m}[r(t) - x_{k}(t)] + F_{k}(t)$$

$$\omega_{k}(s) = (M_{m} + C_{m} + K_{m})[r(s) - x_{k}(s)] + F_{k}(s)$$
(B2)

Then this error is reduced iteratively to achieve a convergence in the solution. A higher controller gain is needed for a desired system response with light damping. This is because for such a system a high overshoot arises and hence a higher controller gain is required to suppress it. The target impedance depends on the structure stiffness as can be seen by rearranging (B2) for zero error (convergence):

$$[\ddot{r}(t) - \ddot{x}_{k}(t)] + 2\omega\zeta[\dot{r}(t) - \dot{x}_{k}(t)] + (\omega^{2} + k)[r(t) - x_{k}(t)] = kr(t)$$
(B3)

Therefore, in the case of a very stiff structure, the target impedance is a lightly damped system, which requires a higher controller gain to guarantee the convergence of the learning impedance system.

Cohen and Flash [9] represented the associative search network (ASN) learning scheme for impedance parameters. This scheme does not use an actuator model or known structure stiffness. It is a stochastic scheme that uses a single scalar value as a measure of the system performance. This method can be used if no information is given about the system dynamics and the structure. However, there is a tradeoff between the time spent on learning and the quality of the outcome of that learning.

Katic and Vukobratovic [23] used the so-called "connectionist architectures" as impedance control and showed a fast and robust on-line learning. The main feature of this scheme is the use of multilayer perceptions with special topology with are integrated in non-learning impedance control algorithms.

B.4. Further control methods

Tzierakis and Koumboulis [57] developed a method for multi manipulator systems, where apart from position control, also the internal forces need to be controlled. A simple and direct algebraic approach is proposed to handle situations where disturbance forces act on the handled objects. It is shown that there exists a linear state feedback law, satisfying the independent force and position control and the decoupling between different actuators.

Sekhavat [43] defined a contact task control as the ability of a manipulator to follow a free space trajectory and make contact with the environment until all the energy of impacts is dissipated and the desired contact force is achieved. They developed a controller for hydraulic actuators to regulate the impacts during transition phase from free space to constraint motion. The scheme does not require force or velocity feedback or knowledge of the structural or hydraulic parameters.

Schaffer and Hirzinger [1] compared impedance, stiffness and admittance control. A new impedance controller enhanced by local stiffness control showed a better performance than classical impedance and stiffness control. Compared to admittance control, it has lower geometric accuracy, but higher bandwidth and impedance range. Pratt [40] showed the relationship between the impedance and the electrical circuits of Norton and Thevenin (Figure B 3). The electrical current is modified passing the circuit and has the effect of a virtual impedance. *B* and *K* command the actuator to create a virtual mechanical impedance equivalent to a parallel combination of a damper and a spring, respectively. *F* commands the actuator to add an offset force in parallel with the damper and spring, or, when divided by the stiffness *K*, an offset position in series with the virtual spring.



Figure B 3: Norton and Thevenin Equivalents of Virtual Impedance [40]

Using Hogan's impedance control theory, Mills [33] developed an open-loop control over generalized forces and position. Open-loop design does not require a force feedback, hence no force load cell is required. However, this open loop control only allows for a crude control over the contact forces. In comparison to the closed loop methods, the open loop approach moreover does not exhibit robustness to dynamic parameter uncertainty. The main feature of this approach is that both the actuator and structural dynamic parameters are known exactly and the mechanical impedance of the manipulator remains unchanged during contact between actuator and structure. The input trajectory to the manipulator is varied, so that the desired force and position trajectories result. The input variables are chosen according to the equation below, where the index *s* is used for the structural mass, damping and stiffness matrices, where \bar{x} is the desired actuator deformation and x^i is the chosen displacement in order to achieve the desired displacement *r*.

$$M(\ddot{r} - \ddot{x}^{i}) + B(\dot{r} - \dot{x}^{i}) + K(r - x^{i}) = -[M_{s}\ddot{x} + B_{s}\dot{x} + K_{s}\bar{x}]$$
(B4)

In order to reject undesirable high-frequency disturbances (caused by a stiff structure), Anderson and Spong [3] introduced a general control approach, called hybrid impedance control (HIC), which in its simplest forms reduces to Hogan's impedance control. It combines impedance control and hybrid position/force control into one strategy while allowing for more sophisticated impedances.

Lawrence [28] showed the effects of computation and communication delays and manipulator dynamics on the behavior of two primary approaches to impedance control. For both position and force control, the stability limits for the impedance parameters are presented.

If the manipulator runs in position control, forces are sensed explicitly and position commands are issued to the inner loop controller. The measured interaction force F allows the creation of the position adjustment vector x_a :

$$x_{a}(s) = [Ms^{2} + Cs + K]^{-1}F(s)$$

$$x_{c} = r + x_{a}$$
(B5)

This control scheme has been applied in this dissertation, where the adjustment vector is equivalent to the feedforward input.

The flexibility in the actuators, as well as the iteration rate of the controller limits the admissible gains and therefore the overall performance. Pelletier and L. K. Daneshmend [39] developed a method based on Whitney's damping control scheme, where the actuator reacts as a generalized damper. It also works if it is not sure if position or force control should be applied. The best solution will always be a compromise between speed of response and stability at high structure stiffness. The aim is therefore to adjust the system parameters to tune the response according to the stiffness. Typically if the non-adaptive robot hits a soft wall at a certain initial velocity, it will loose speed slowly till it reaches the desired force. This behavior is sluggish from a force control point of view but reacts according to the desired impedance. On the other hand, with the proposed adaptive damping control scheme, the controller will speed up the force response and increase the robot velocity to reach the desired force faster. For low structure stiffness, the adapter speeds up the

response, while for high stiffness, it has a stabilizing effect that allows the controller to be stable at a stiffness four times higher than in the non-adaptive case.

Lasky and Hsia [27] presented a reference force-tracking impedance control system, consisting of a conventional impedance controller in the inner-loop and a trajectory-modifying controller in the outer-loop for force tracking. Like shown earlier in position controlled actuator systems the interaction force is controlled indirectly through an impedance function. The outer-loop trajectory modification algorithm adds the capability of reference force tracking to the inner-loop impedance control.

Liu and Goldenberg [29] controlled the impedance by tracking a desired acceleration trajectory. Defining a desired acceleration trajectory online and the use of a PI feedback controller allowed an efficient and robust impedance control.

Task performance depends on the accuracy at which the desired impedance is attained. The conflict between impedance accuracy and robustness to uncertainties has been presented by Valency and Zacksenhouse [58] by an Eigenvalue analysis. Three approaches have been suggested to enhance the robustness of impedance control, which are low-order impedance control strategies, such as stiffness control, adaptive and robust control methods and the inner/outer control strategy. The inner/outer method achieves its robustness at the expense of accurate tracking of the desired impedance. The problem is that the impedance adaptive methods do not account for external disturbances. Valency and Zacksenhouse showed a model with improved robustness and accurate impedance tracking. The proposed method takes advantage of the error-correction capabilities of position controllers while maintaining good impedance tracking.

C.APPENDIX – Transfer functions for EM actuator

C.1. Open loop transfer functions

The actuator in open loop is a two input system. The actuator displacement depends on the voltage input *u* and the interaction force *F*, so that $x = H_{ux}u - H_{fx}F$. The transfer function representation of the state space description of the electromagnetic actuator in equation (3.19) is:

$$H_{ux} = \frac{Bl}{mLs^{2} + (mR + cL)s + cL + (Bl)^{2}} \frac{1}{s}$$

$$H_{fx} = \frac{R + Ls}{Bl} H_{ux}$$
(C1)

Assuming a linear structure stiffness, the interaction force is F = kx. The actuator can be considered as a one-input system, $x = \frac{H_{ux}}{1 + kH_{fx}}u = H_{ol}u$, where H_{ol} is the linearized open loop transfer function relating the actuator displacement to the

applied input voltage,

$$H_{ol} = \frac{x}{u} = \frac{Bl}{mLs^3 + (mR + cL)s^2 + (cL + (Bl)^2 + kL)s + kR}$$
(C2)

C.2. Closed loop transfer functions

Applying closed loop displacement control with a proportional controller, the voltage input is the displacement error multiplied by the proportional gain K_p .

Substituting $u = K_p(r-x)$ into $x = H_{ux}u - H_{fx}F$ yields the actuator displacement in closed loop control for an interaction force *F*.

$$x = \frac{H_{ux}K_p}{1 + H_{ux}K_p}r - \frac{H_{fx}}{1 + H_{ux}K_p}F$$
(C3)

In order to derive the actuator impedance, an additional external force disturbance on the system is considered (Figure 4.1). With a linear structure stiffness, and the external force disturbance, the interface force $F = kx + F_{ext}$, and the actuator displacement from equation (C3) is given by

$$x = \frac{Bl K_p H_{ux}}{(Bl K_p + k(R+Ls))H_{ux} + Bl} r - \frac{(R+Ls)H_{ux}}{(Bl K_p + k(R+Ls))H_{ux} + Bl} F_{ext}$$

$$= H_{cl}r - \frac{R+Ls}{Bl K_p} H_{cl}F_{ext}$$
(C4)

The linearized closed loop transfer function relates the actuator displacement to the reference displacement input.

$$H_{cl} = \frac{x}{r} = \frac{Bl K_p}{mLs^3 + (mR + cL)s^2 + (cL + (Bl)^2 + kL)s + Rk + Bl K_p}$$
(C5)

The second term in equation (C4) is the compliance of the coupled system and determines the deformation of the coupled system due to the external force. The reciprocal of the compliance is the impedance.

$$\hat{K} = \left| \frac{x_{ext}}{F_{ext}} \right| = \frac{Bl K_p}{(R+Ls)H_{cl}} = \frac{mLs^3 + (mR+cL)s^2 + (cL+(Bl)^2 + kL)s + Rk + Bl K_p}{R+Ls}$$

$$= \underbrace{\frac{mLs^3 + (mR+cL)s^2 + (cL+(Bl)^2)s + Bl K_p}{R+Ls}}_{\text{actuator impedance}} + k$$
(C6)

C.3. Feedforward

Now, the reference input r is altered by an additional feedforward input. Substituting

$$r + \frac{R}{Bl K_p} F$$
 for r in equation (C3) gives

$$x = \frac{Bl K_{p} r - LsF}{mLs^{3} + (mR + cL)s^{2} + (cL + (Bl)^{2})s + Bl K_{p}}$$
(C7)

Substituting again $F = kx + F_{ext}$ into equation (C7) gives

$$x = \frac{Bl K_p H_{ux}}{(Bl K_p + kLs)H_{ux} + Bl} r - \frac{(R + Ls)H_{ux}}{(Bl K_p + kLs)H_{ux} + Bl} F_{ext}$$

$$= H_{cl,FF} r - \frac{R + Ls}{Bl K_p} H_{cl,FF} F_{ext}$$
(C8)

The linearized and compensated closed loop transfer function relating the actuator displacement to the reference displacement then is different from equation (C5) and is given by

$$H_{cl,FF} = \frac{Bl K_{p}}{mLs^{3} + (mR + cL)s^{2} + (cL + (Bl)^{2} + kL)s + Bl K_{p}}$$
(C9)

The impedance of the compensated system \hat{K}_{FF} is now lowered in comparison to equation (C6) by an additional term.

$$\hat{K}_{FF} = \left| \frac{x_{ext}}{F_{ext}} \right| = \frac{Bl K_p}{(R+Ls) H_{cl,FF}} = \frac{mLs^3 + (mR+cL)s^2 + (cL+(Bl)^2 + kL)s + Bl K_p}{R+Ls}$$
(C10)
=
$$\frac{mLs^3 + (mR+cL)s^2 + (cL+(Bl)^2)s + Bl K_p}{R+Ls} - \frac{R}{R+Ls}k + k = \hat{K} - \frac{R}{R+Ls}k$$

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